

# ***Bounded Model Checking of Multi-threaded Software using SMT solvers***

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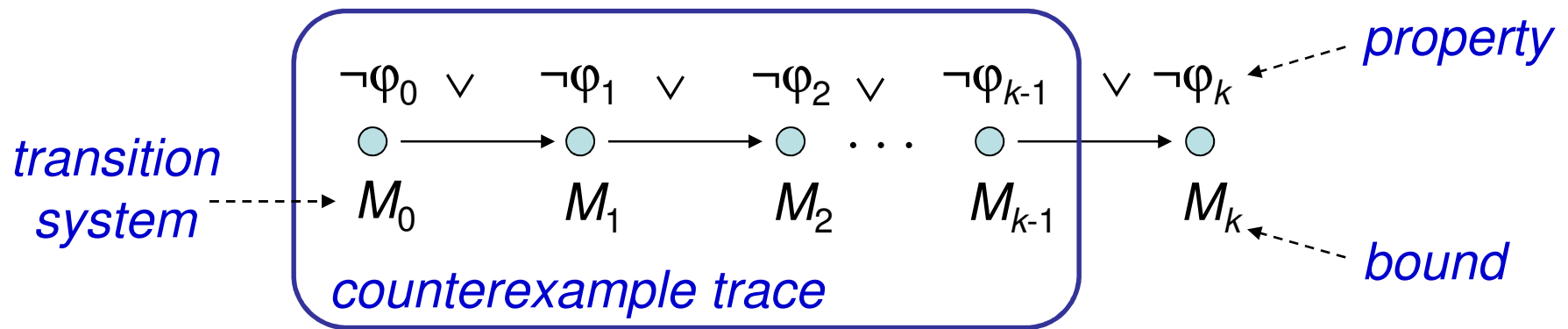
# *Bounded Model Checking of Multi-threaded Software using SMT solvers*

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# Bounded Model Checking (BMC)

Basic Idea: check negation of given property up to given depth



- transition system  $M$  unrolled  $k$  times
  - for programs: unroll loops, unfold arrays, ...
- translated into verification condition  $\psi$  such that
  - $\psi$  satisfiable iff  $\varphi$  has counterexample of max. depth  $k$**
- has been applied successfully to verify (embedded) software

# BMC of Multi-threaded Software

- concurrency bugs are tricky to **reproduce/debug** because they usually occur under specific thread interleavings
  - most common errors: *67% related to atomicity and order violations, 30% related to deadlock* [Lu et al.'08]
- problem: the number of interleavings grows exponentially with the number of threads ( $n$ ) and program statements ( $s$ )
  - *number of executions:  $O(n^s)$*
  - *context switches among threads increase the number of possible executions*
- two important observations help us:
  - concurrency bugs are shallow [Qadeer&Rehof'05]
  - SAT/SMT solvers produce unsatisfiable cores that allow us to remove logic that is not relevant

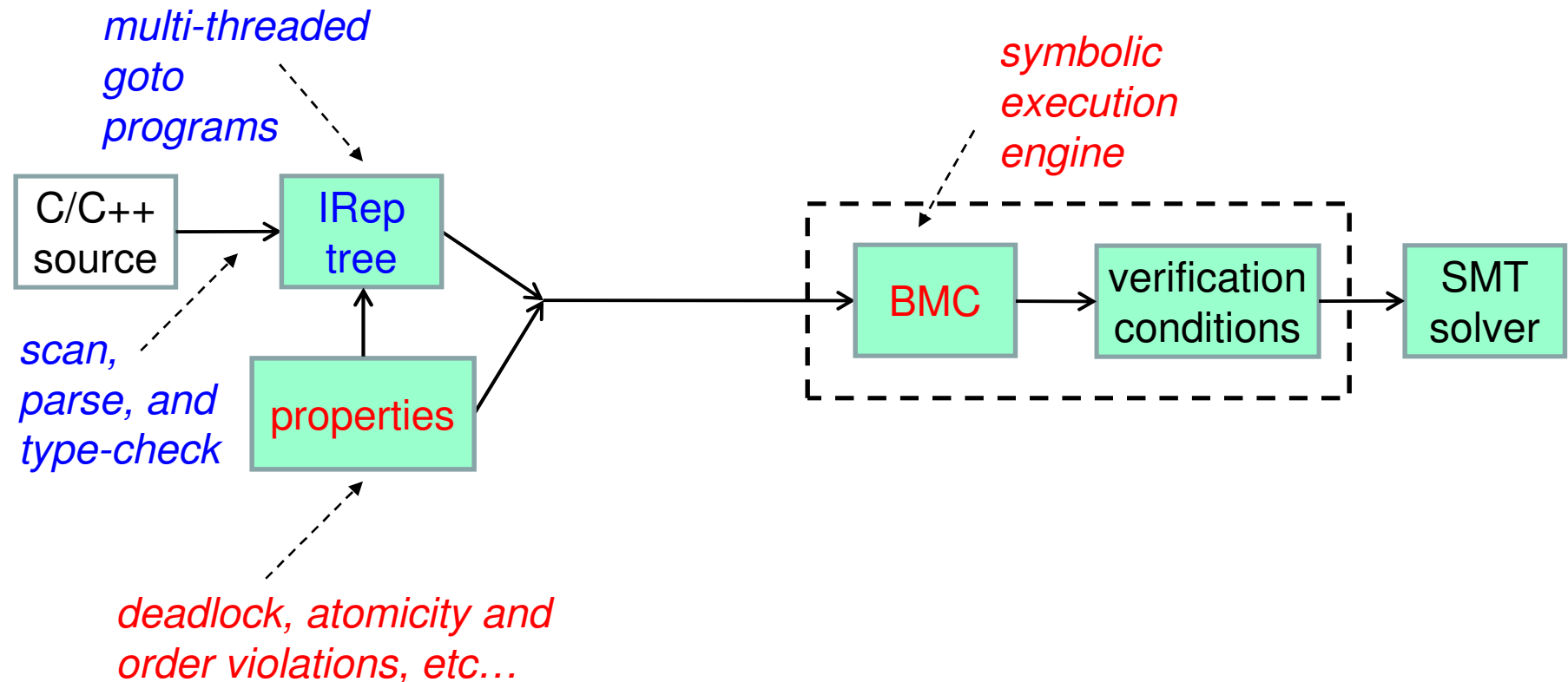
# Objective of this work

## Exploit SMT to improve BMC of multi-threaded software

- exploit SMT solvers to:
  - prune the *property and data dependent* search space (non-chronological backtracking and conflict clauses learning)
  - remove interleavings that are not relevant by analyzing the proof of unsatisfiability [**NOT** Craig Interpolants **yet**]
- propose three approaches to SMT-based BMC:
  - *lazy exploration* of the interleavings
  - *schedule guards* to encode all interleavings
  - *underapproximation and widening (UW)* [Grumberg&et al.'05]
- evaluate our approaches over multi-threaded applications

# Lazy exploration of interleavings

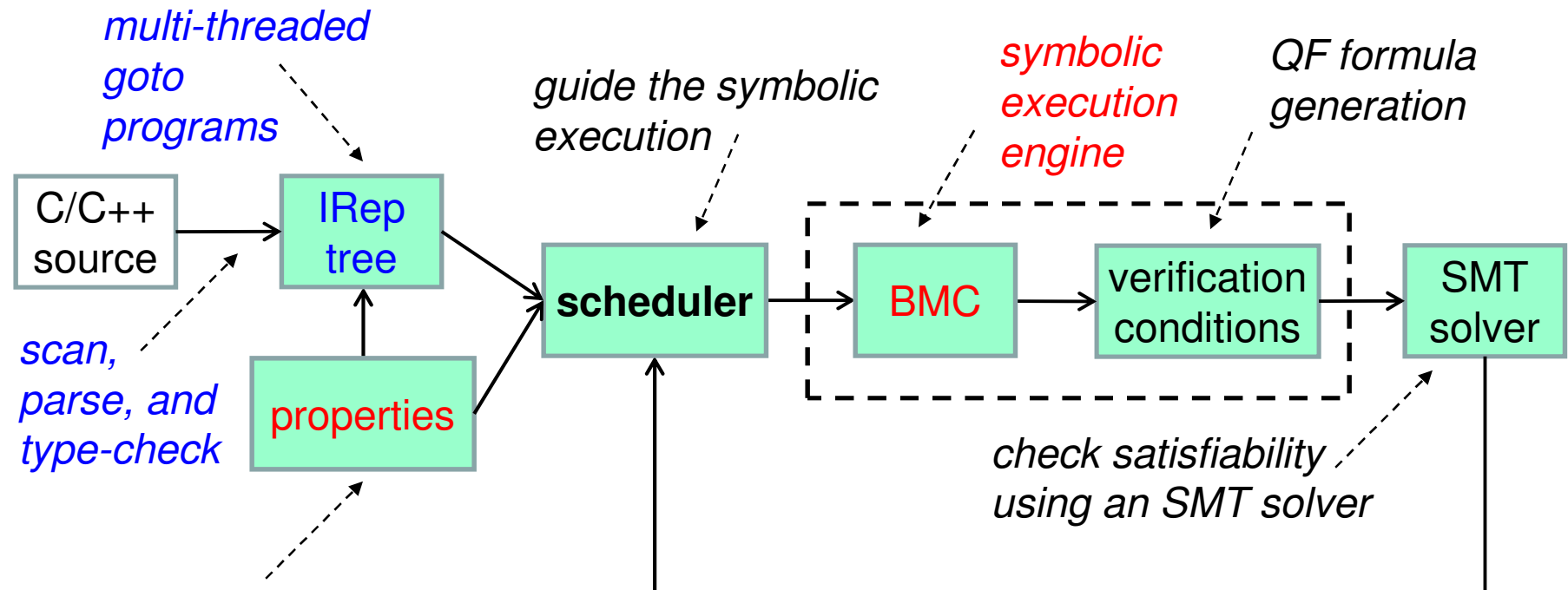
**Idea: iteratively generate all possible interleavings and call the BMC procedure on each interleaving**



reused/extended from the  
CBMC model checker

# Lazy exploration of interleavings

**Idea: iteratively generate all possible interleavings and call the BMC procedure on each interleaving**



**reused/extended** from the  
 CBMC model checker

# Lazy exploration: Scheduler

- the scheduler (emulate the

Step 1: expand thread with smallest subtree

Step 2: generate interleavings starting with  $T_2$

Thread  $T_1$     Thread  $T_2$

$a_1$

$b_1$

$a_2$

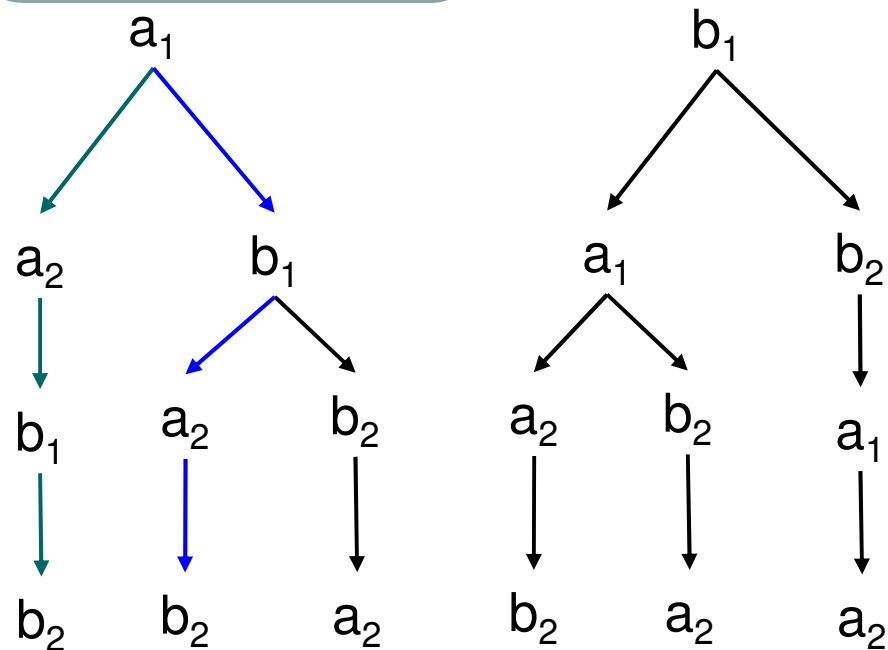
$b_2$

Thread interleavings:

$a_1; a_2; b_1; b_2$

$a_1; b_1; a_2; b_2$

...



- allow preemptions only before visible statements (global variables and synchronization points)



# Running Example

- the program has sequences of operations that need to be protected together to avoid atomicity violation
  - requirement: the region of code (*val1* and *val2*) should execute atomically

```
Thread twoStage  
1: lock(m1);  
2: val1 = 1;  
3: unlock(m1);  
4: lock(m2);  
5: val2 = val1 + 1;  
6: unlock(m2);
```

A state  $s \in S$  consists of the value of the program counter  $pc$  and the values of all program variables

*program counter: 0*  
*mutexes: m1=0; m2=0;*  
*global variables: val1=0; val2=0;*  
*local variables: t1= -1; t2= -1;*

```
7: t1 = val1;  
8: lock(m1);  
9: t2 = val2;  
10: unlock(m1);  
11: lock(m2);  
12: t2 = val2;  
13: lock(m2);  
14: t2 = val2;  
15: unlock(m2);  
16: assert(t2==(t1+1));
```

# Lazy exploration: interleaving $I_s$

statements:

val1-access:

val2-access:

```
Thread twoStage
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

```
Thread reader
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2 == (t1 + 1));
```

*program counter: 0*

*mutexes: m1=0; m2=0;*

*global variables: val1=0; val2=0;*

*local variables: t1= -1; t2= -1;*

# Lazy exploration: interleaving $I_s$

statements: 1

val1-access:

val2-access:

● Thread twoStage  
1: lock(m1);  
2: val1 = 1;  
3: unlock(m1);  
4: lock(m2);  
5: val2 = val1 + 1;  
6: unlock(m2);

Thread reader  
7: lock(m1);  
8: if (val1 == 0) {  
9: unlock(m1);  
10: return NULL; }  
11: t1 = val1;  
12: unlock(m1);  
13: lock(m2);  
14: t2 = val2;  
15: unlock(m2);  
16: assert(t2 == (t1 + 1));

**program counter: 1**

*mutexes: m1=1; m2=0;*

*global variables: val1=0; val2=0;*

*local variables: t1= -1; t2= -1;*

# Lazy exploration: interleaving $I_s$

statements: 1-2

val1-access:  $W_{twoStage,2}$

val2-access:

write access to the shared variable *val1* in statement 2 of the thread *twoStage*

```

Thread twoStage
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
  
```

```

Thread reader
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
  
```

**program counter: 2**

mutexes:  $m1=1; m2=0;$

global variables: **val1=1**;  $val2=0;$

local variables:  $t1= -1; t2= -1;$

# Lazy exploration: interleaving $I_s$

statements: 1-2-3

val1-access:  $W_{\text{twoStage},2}$

val2-access:

```
Thread twoStage
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```



```
Thread reader
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

**program counter: 3**

*mutexes: **m1=0**; m2=0;*

*global variables: val1=1; val2=0;*

*local variables: t1= -1; t2= -1;*

# Lazy exploration: interleaving $I_s$

statements: 1-2-3-7

val1-access:  $W_{\text{twoStage},2}$

val2-access:

```
Thread twoStage
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

CS1

```
Thread reader
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10: return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

**program counter: 7**

**mutexes: m1=1; m2=0;**

**global variables: val1=1; val2=0;**

**local variables: t1= -1; t2= -1;**

# Lazy exploration: interleaving I<sub>c</sub>

statements: 1-2-3-7-8

val1-access:  $W_{twoStage,2} - R_{reader,8}$

val2-access:

read access to the shared variable *val1* in statement 8 of the thread *reader*

```
Thread twoStage
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

CS1

```
Thread reader
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

**program counter: 8**  
*mutexes: m1=1; m2=0;*  
*global variables: val1=1; val2=0;*  
*local variables: t1= -1; t2= -1;*

# Lazy exploration: interleaving $I_s$

statements: 1-2-3-7-8-11

val1-access:  $W_{\text{twoStage},2}$  -  $R_{\text{reader},8}$  -  $R_{\text{reader},11}$

val2-access:

```
Thread twoStage
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

CS1

```
Thread reader
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2 == (t1 + 1));
```

**program counter: 11**

*mutexes: m1=1; m2=0;*

*global variables: val1=1; val2=0;*

*local variables: t1= 1; t2= -1;*



# Lazy exploration: interleaving $I_s$

statements: 1-2-3-7-8-11-12

val1-access:  $W_{twoStage,2}$  -  $R_{reader,8}$  -  $R_{reader,11}$

val2-access:

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

CS1

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2 == (t1 + 1));
```

**program counter: 12**

*mutexes: m1=0; m2=0;*

*global variables: val1=1; val2=0;*

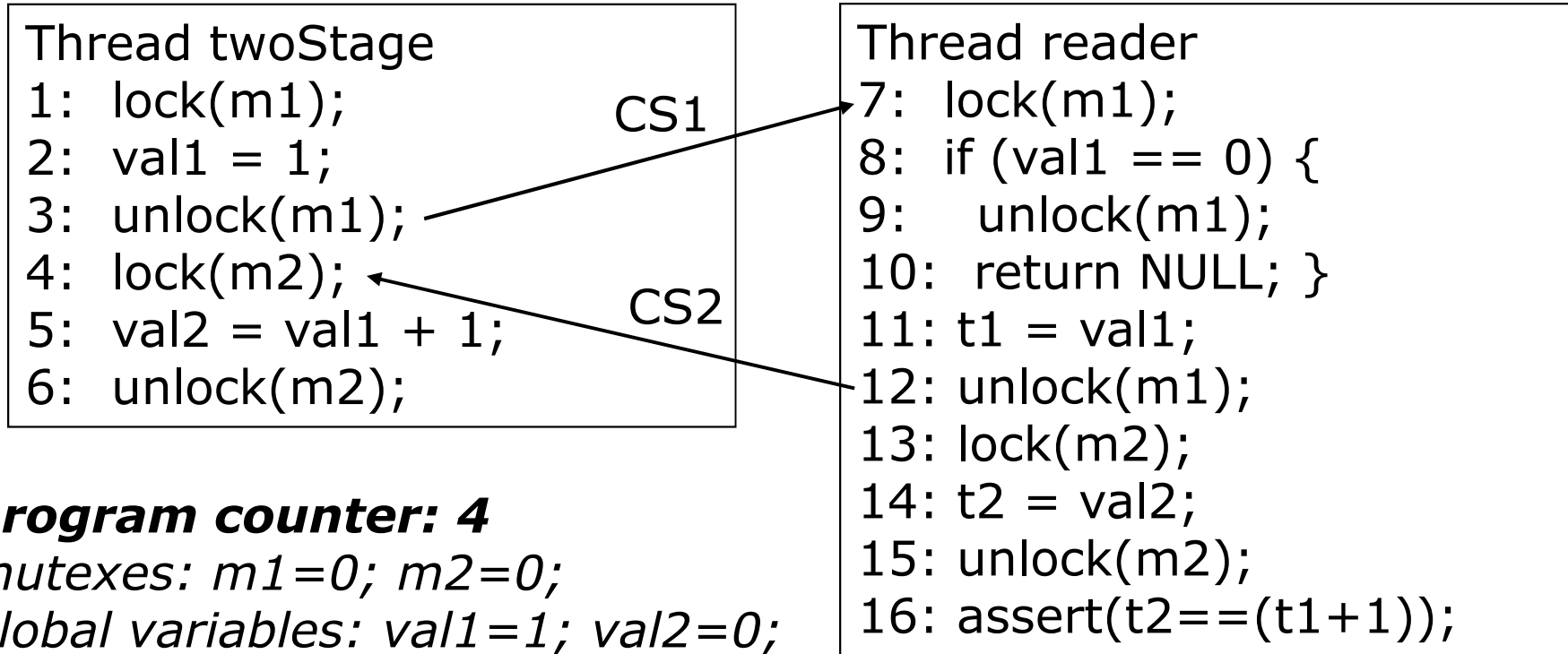
*local variables: t1= 1; t2= -1;*

# Lazy exploration: interleaving $I_s$

statements: 1-2-3-7-8-11-12

val1-access:  $W_{twoStage,2}$  -  $R_{reader,8}$  -  $R_{reader,11}$

val2-access:



**program counter: 4**

*mutexes: m1=0; m2=0;*

*global variables: val1=1; val2=0;*

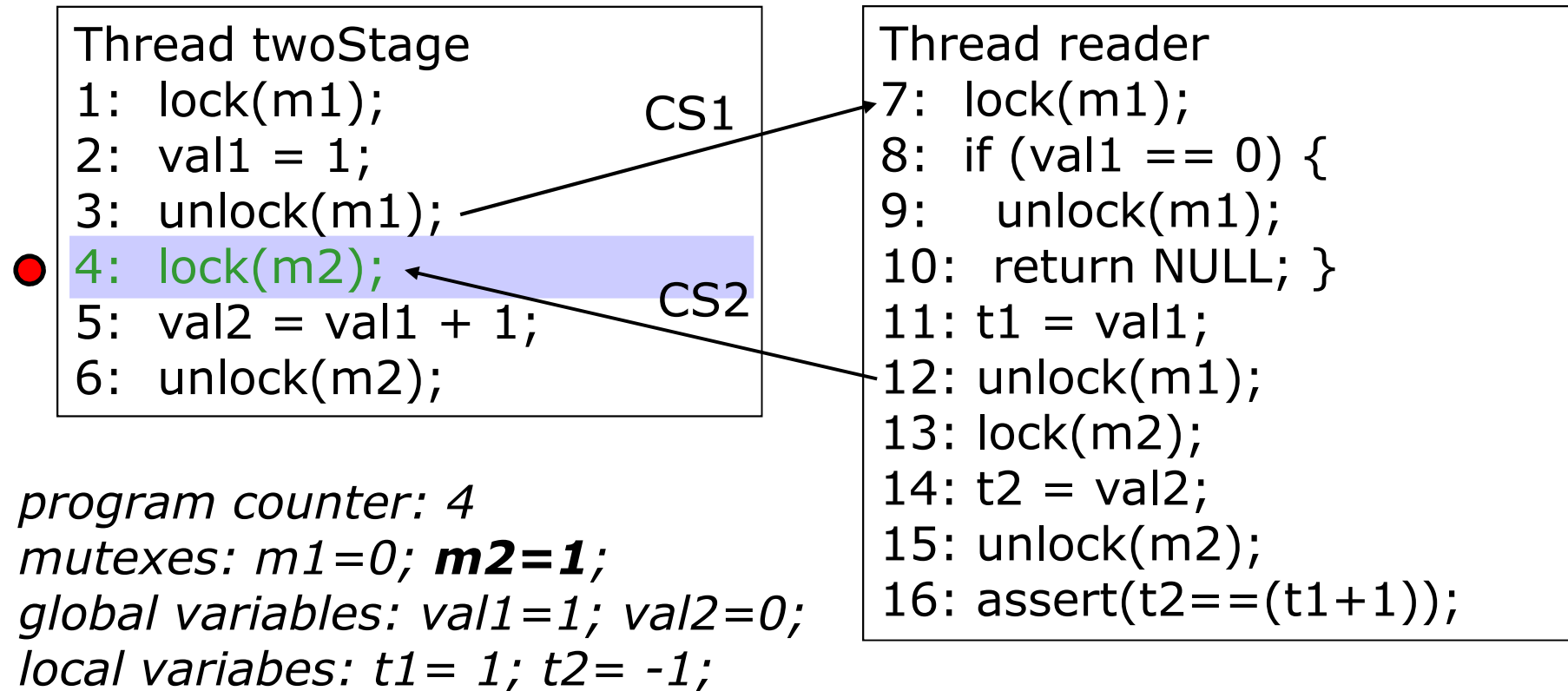
*local variables: t1= 1; t2= -1;*

# Lazy exploration: interleaving $I_s$

statements: 1-2-3-7-8-11-12-4

val1-access:  $W_{twoStage,2}$  -  $R_{reader,8}$  -  $R_{reader,11}$

val2-access:

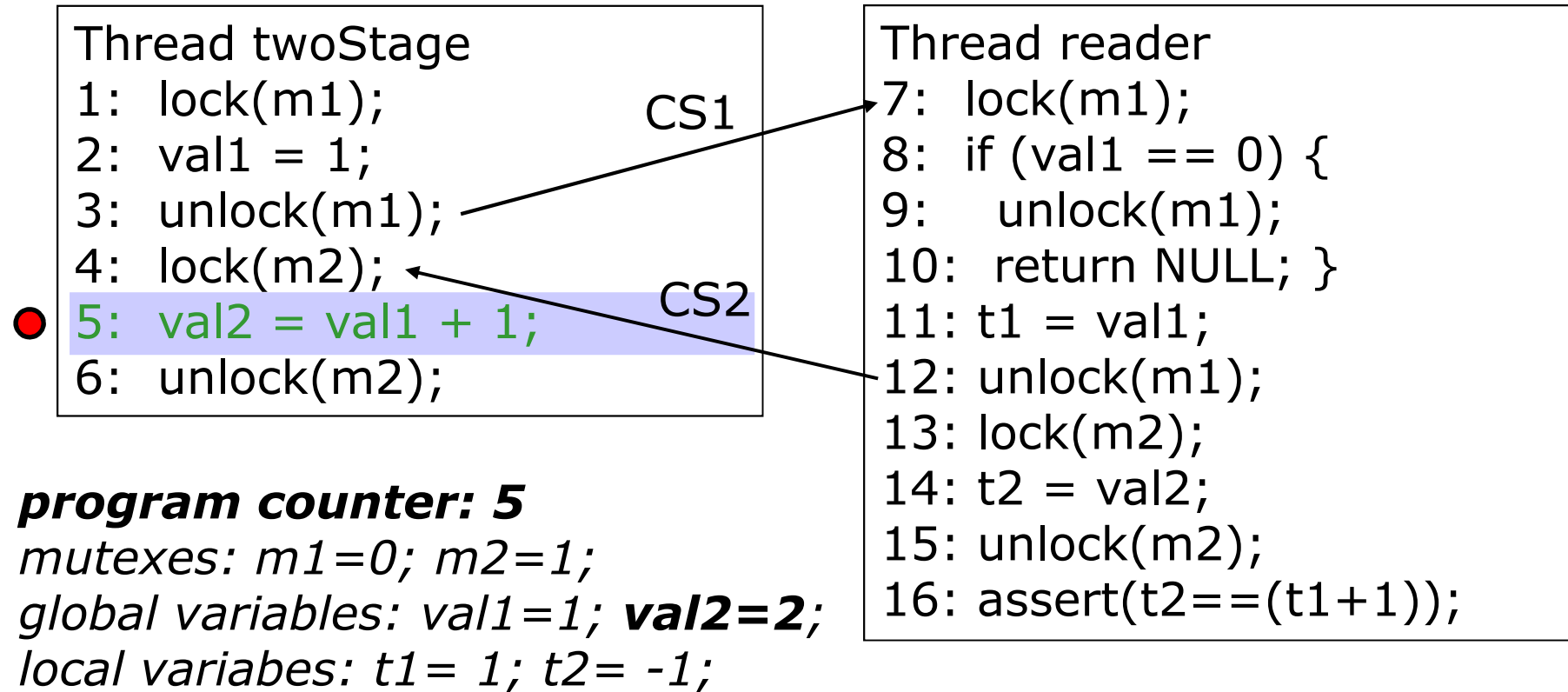


# Lazy exploration: interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5

val1-access:  $W_{\text{twoStage},2}$  -  $R_{\text{reader},8}$  -  $R_{\text{reader},11}$  -  $R_{\text{twoStage},5}$

val2-access:  $W_{\text{twoStage},5}$



# Lazy exploration: interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5-6

val1-access:  $W_{twoStage,2}$  -  $R_{reader,8}$  -  $R_{reader,11}$  -  $R_{twoStage,5}$

val2-access:  $W_{twoStage,5}$

Thread twoStage

```

1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
  
```

CS1

CS2

Thread reader

```

7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2 == (t1 + 1));
  
```



**program counter: 6**

*mutexes: m1=0; m2=0;*

*global variables: val1=1; val2=2;*

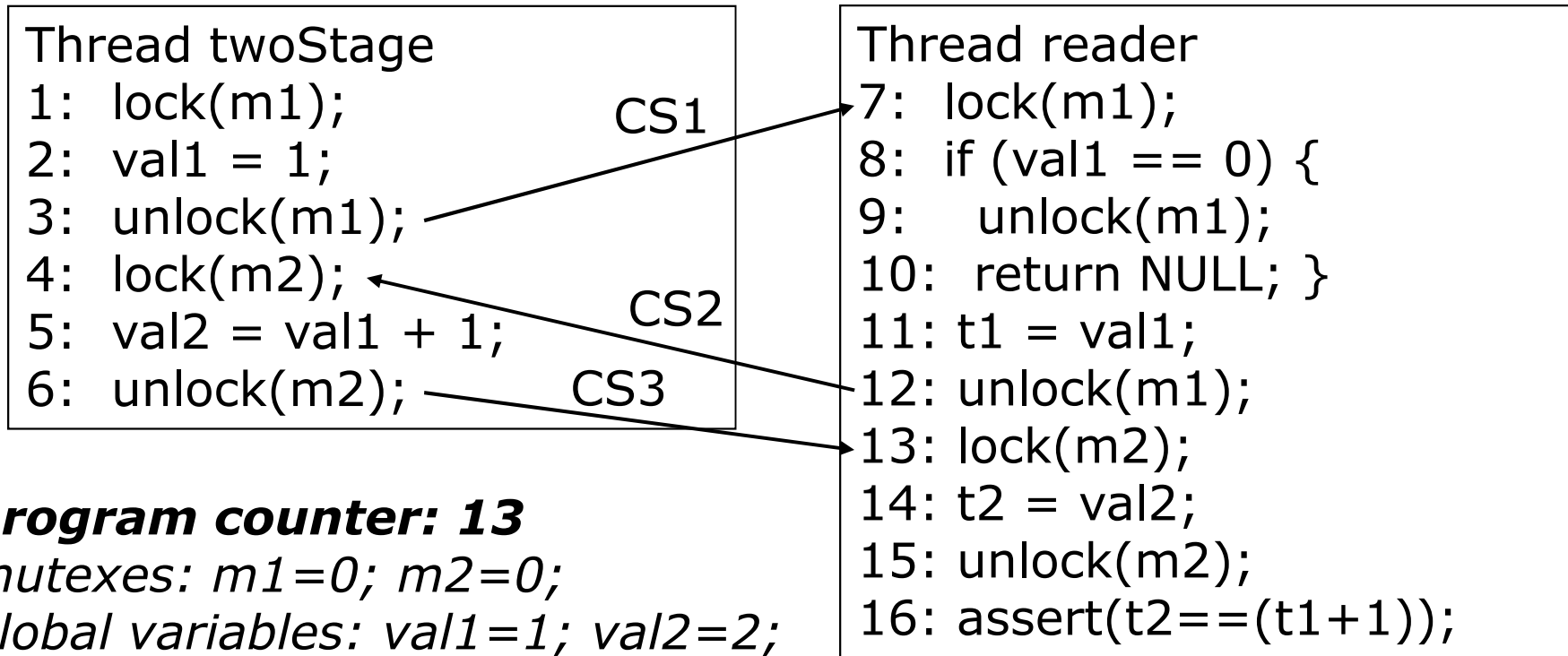
*local variables: t1= 1; t2= -1;*

# Lazy exploration: interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5-6

val1-access:  $W_{twoStage,2}$  -  $R_{reader,8}$  -  $R_{reader,11}$  -  $R_{twoStage,5}$

val2-access:  $W_{twoStage,5}$



**program counter: 13**

*mutexes: m1=0; m2=0;*

*global variables: val1=1; val2=2;*

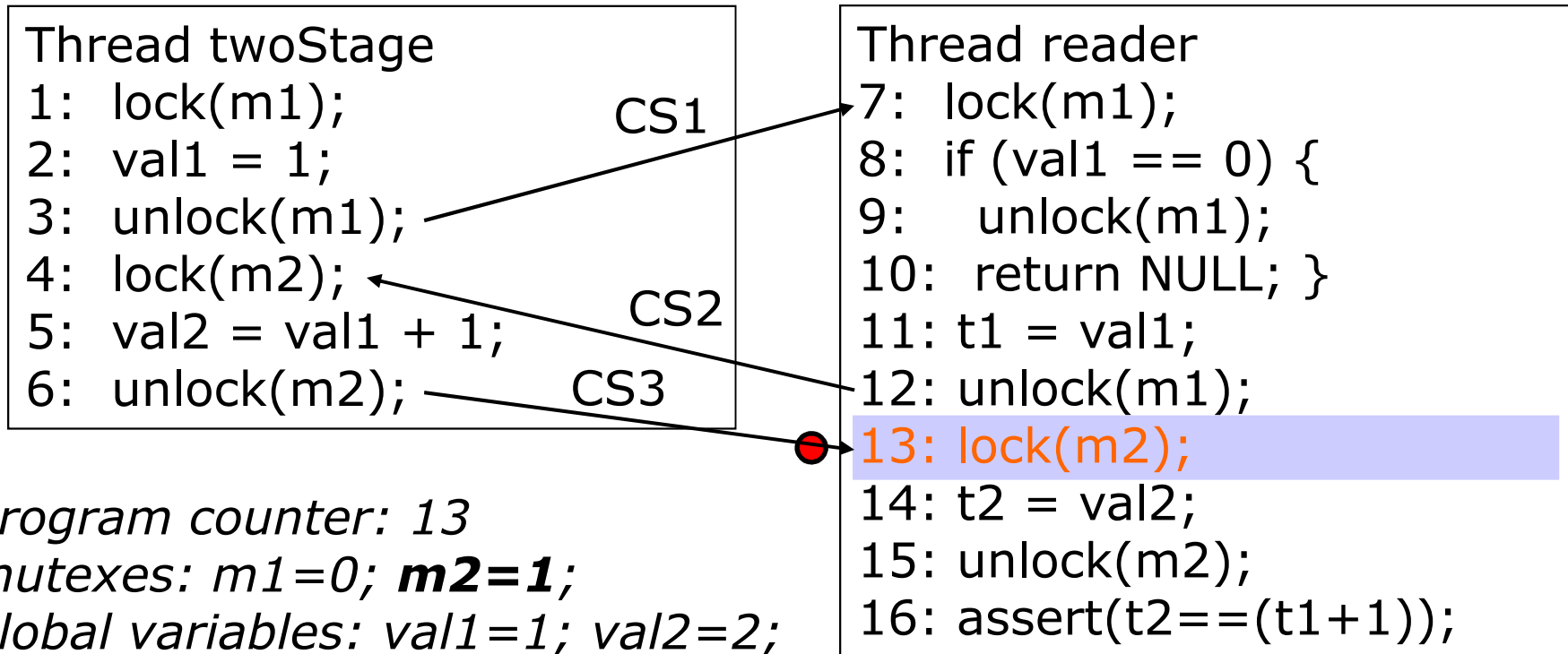
*local variables: t1= 1; t2= -1;*

# Lazy exploration: interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5-6-13

val1-access:  $W_{twoStage,2}$  -  $R_{reader,8}$  -  $R_{reader,11}$  -  $R_{twoStage,5}$

val2-access:  $W_{twoStage,5}$



*program counter: 13*

*mutexes: m1=0; **m2=1**;*

*global variables: val1=1; val2=2;*

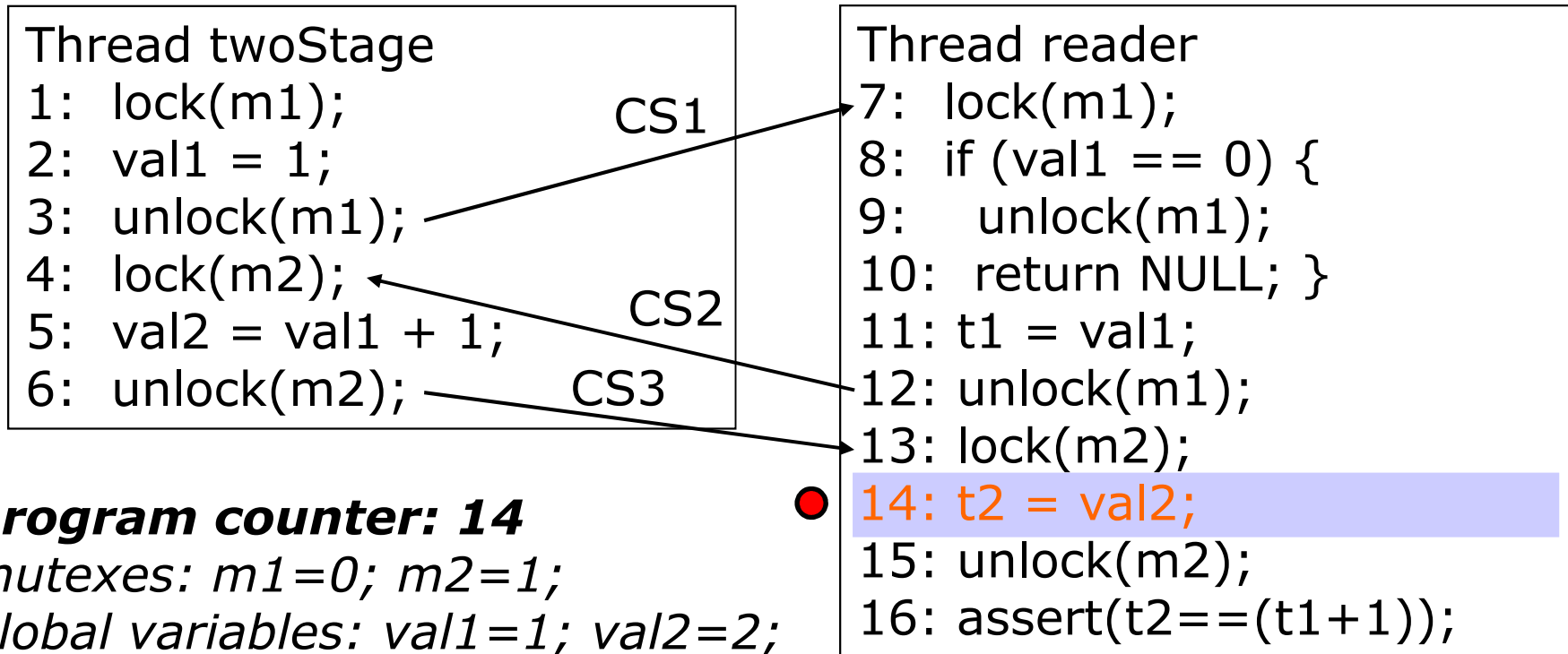
*local variables: t1= 1; t2= -1;*

# Lazy exploration: interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5-6-13-14

val1-access:  $W_{\text{twoStage},2}$  -  $R_{\text{reader},8}$  -  $R_{\text{reader},11}$  -  $R_{\text{twoStage},5}$

val2-access:  $W_{\text{twoStage},5}$  -  $R_{\text{reader},14}$



**program counter: 14**

mutexes: m1=0; m2=1;

global variables: val1=1; val2=2;

local variables: t1= 1; **t2= 2;**

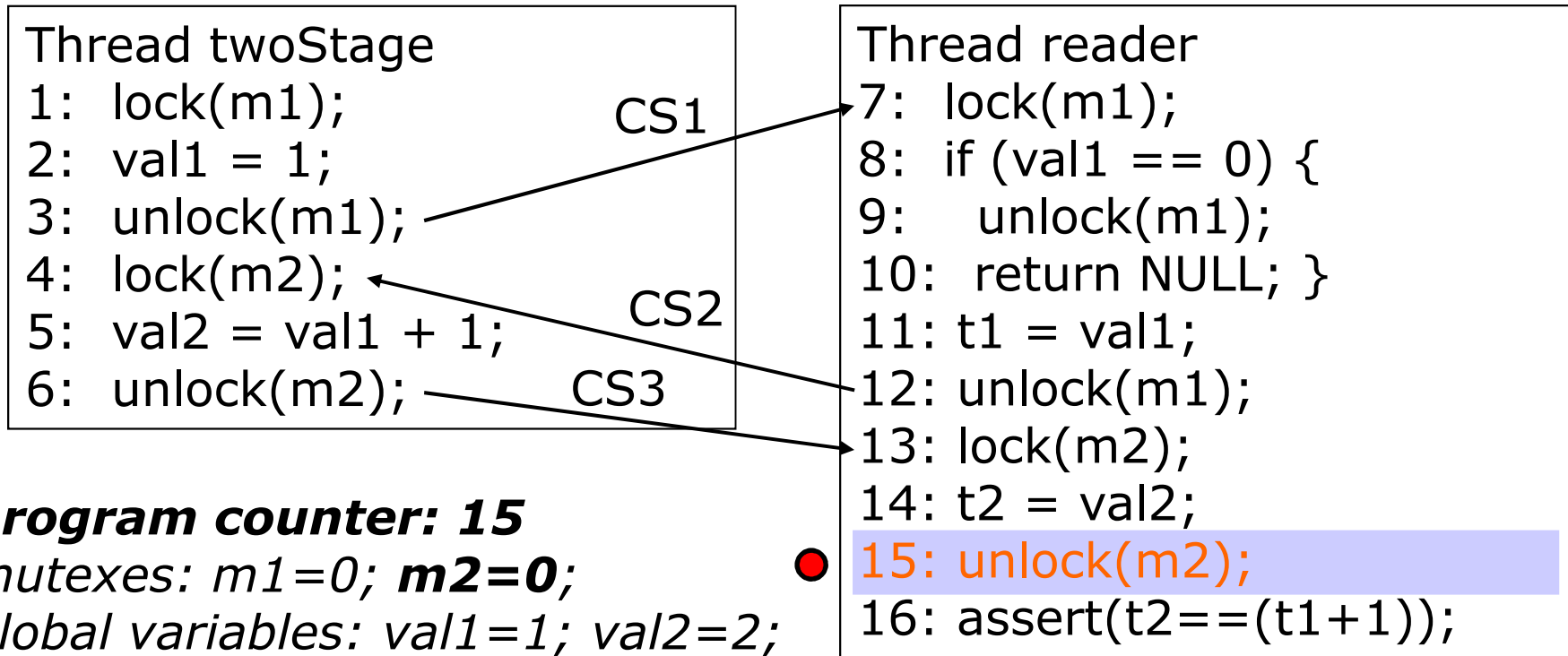


# Lazy exploration: interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15

val1-access:  $W_{twoStage,2}$  -  $R_{reader,8}$  -  $R_{reader,11}$  -  $R_{twoStage,5}$

val2-access:  $W_{twoStage,5}$  -  $R_{reader,14}$



**program counter: 15**

mutexes:  $m1=0$ ;  **$m2=0$** ;

global variables:  $val1=1$ ;  $val2=2$ ;

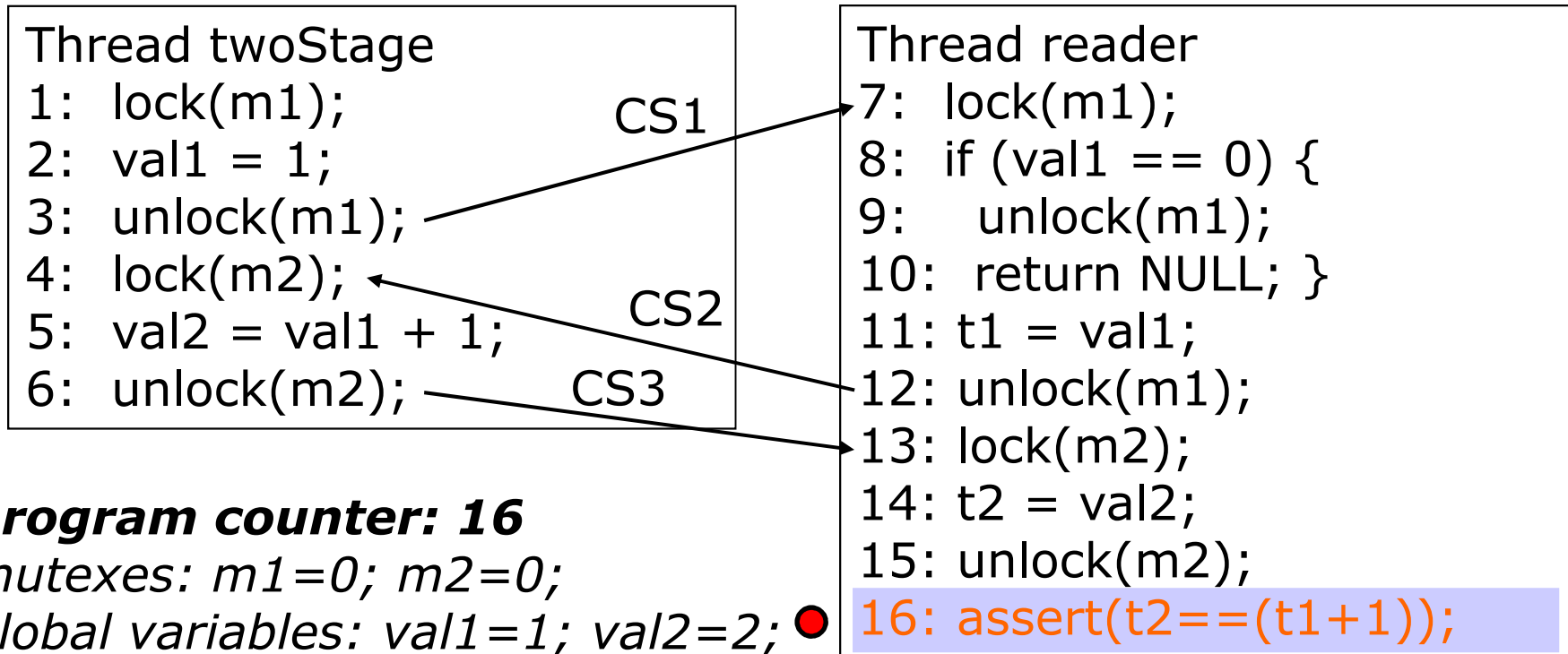
local variables:  $t1=1$ ;  $t2=2$ ;

# Lazy exploration: interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15-16

val1-access:  $W_{twoStage,2}$  -  $R_{reader,8}$  -  $R_{reader,11}$  -  $R_{twoStage,5}$

val2-access:  $W_{twoStage,5}$  -  $R_{reader,14}$



**program counter: 16**

mutexes: m1=0; m2=0;

global variables: val1=1; val2=2; ●

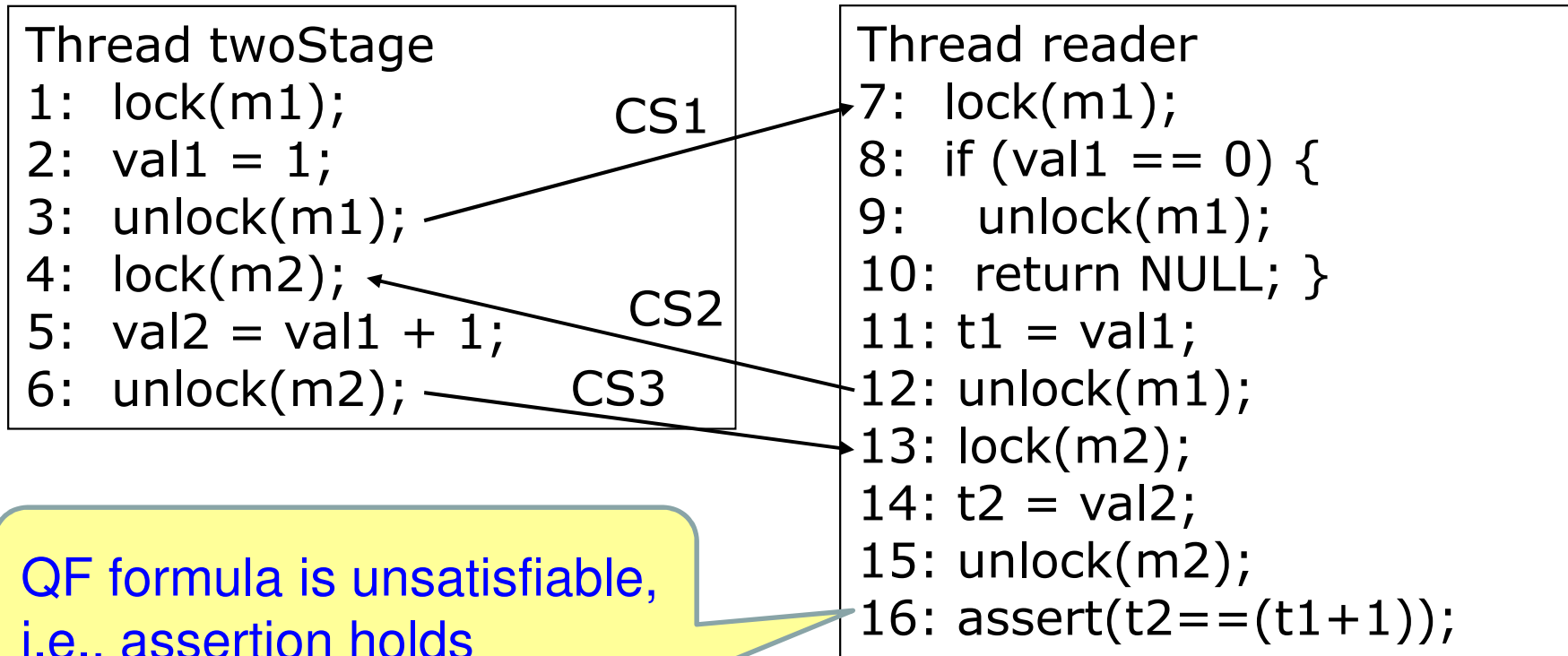
local variables: t1= 1; t2= 2;

# Lazy exploration: interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15-16

val1-access:  $W_{\text{twoStage},2}$  -  $R_{\text{reader},8}$  -  $R_{\text{reader},11}$  -  $R_{\text{twoStage},5}$

val2-access:  $W_{\text{twoStage},5}$  -  $R_{\text{reader},14}$



QF formula is unsatisfiable,  
 i.e., assertion holds

# Lazy exploration: interleaving $I_f$

statements:

val1-access:

val2-access:

```
Thread twoStage
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

```
Thread reader
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2 == (t1 + 1));
```

*program counter: 0*

*mutexes: m1=0; m2=0;*

*global variables: val1=0; val2=0;*

*local variables: t1= -1; t2= -1;*

# Lazy exploration: interleaving $I_f$

statements: 1-2-3

val1-access:  $W_{\text{twoStage},2}$

val2-access:

```
Thread twoStage
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

```
Thread reader
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

**program counter: 3**

*mutexes: m1=0; m2=0;*

*global variables: **val1=1**; val2=0;*

*local variables: t1= -1; t2= -1;*

# Lazy exploration: interleaving $I_f$

statements: 1-2-3

val1-access:  $W_{\text{twoStage},2}$

val2-access:

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

CS1

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2 == (t1 + 1));
```

**program counter: 7**

*mutexes: m1=0; m2=0;*

*global variables: val1=1; val2=0;*

*local variables: t1= -1; t2= -1;*

# Lazy exploration: interleaving $I_f$

statements: 1-2-3-7-8-11-12-13-14-15-16

val1-access:  $W_{\text{twoStage},2}$  -  $R_{\text{reader},8}$  -  $R_{\text{reader},11}$

val2-access:  $R_{\text{reader},14}$

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

CS1

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2 == (t1 + 1));
```

**program counter: 16**

*mutexes: m1=0; m2=0;*

*global variables: val1=1; val2=0;*

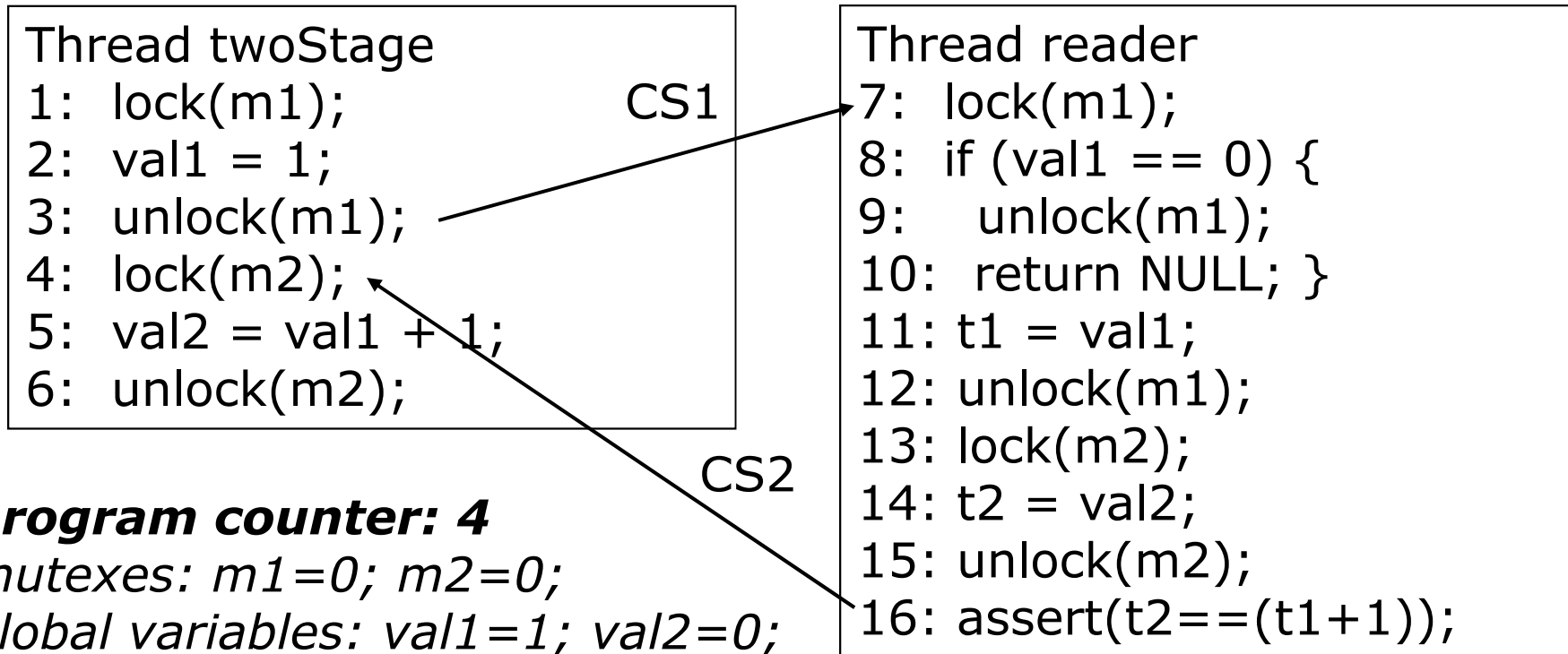
*local variables: t1= 1; t2= 0;*

# Lazy exploration: interleaving $I_f$

statements: 1-2-3-7-8-11-12-13-14-15-16

val1-access:  $W_{\text{twoStage},2}$  -  $R_{\text{reader},8}$  -  $R_{\text{reader},11}$

val2-access:  $R_{\text{reader},14}$



**program counter: 4**

*mutexes: m1=0; m2=0;*

*global variables: val1=1; val2=0;*

*local variables: t1= 1; t2= 0;*

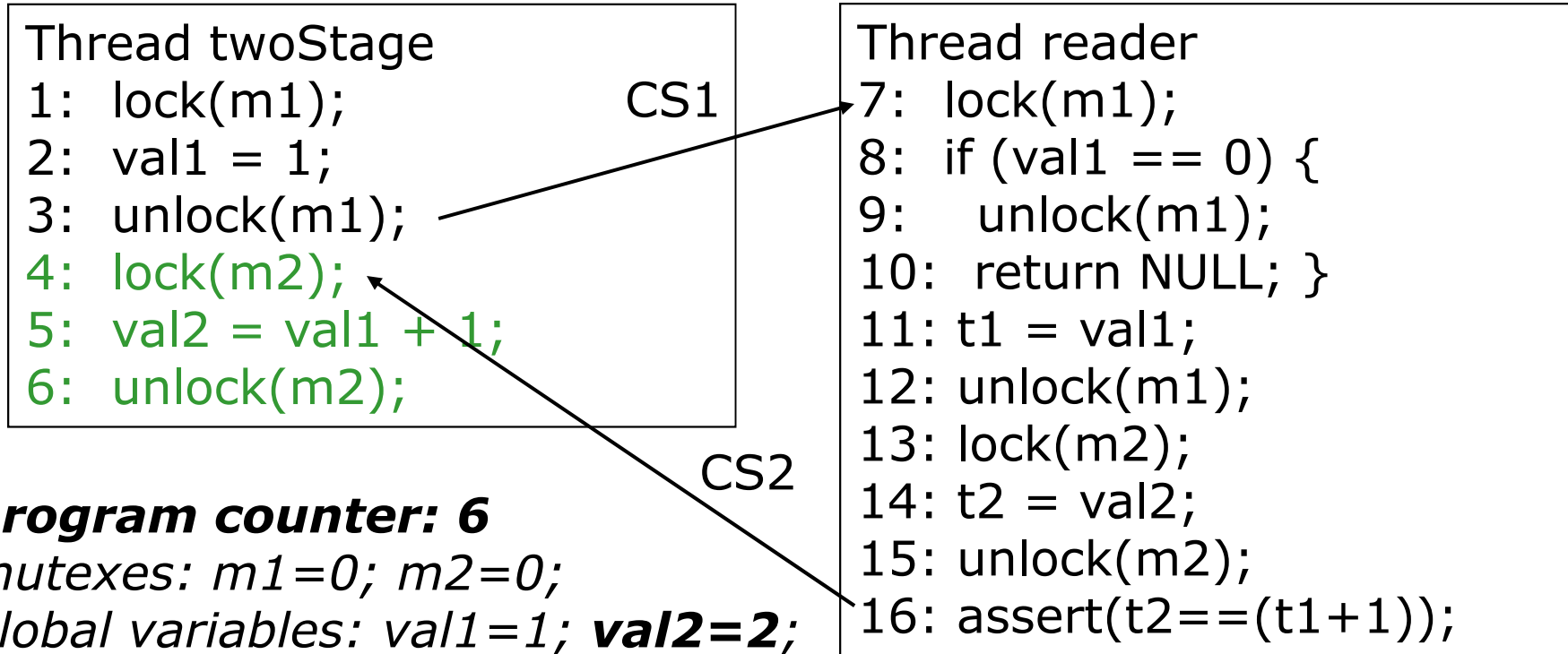


# Lazy exploration: interleaving $I_f$

statements: 1-2-3-7-8-11-12-13-14-15-16-4-5-6

val1-access:  $W_{twoStage,2}$  -  $R_{reader,8}$  -  $R_{reader,11}$  -  $R_{twoStage,5}$

val2-access:  $R_{reader,14}$  -  $W_{twoStage,5}$



**program counter: 6**

mutexes: m1=0; m2=0;

global variables: val1=1; **val2=2;**

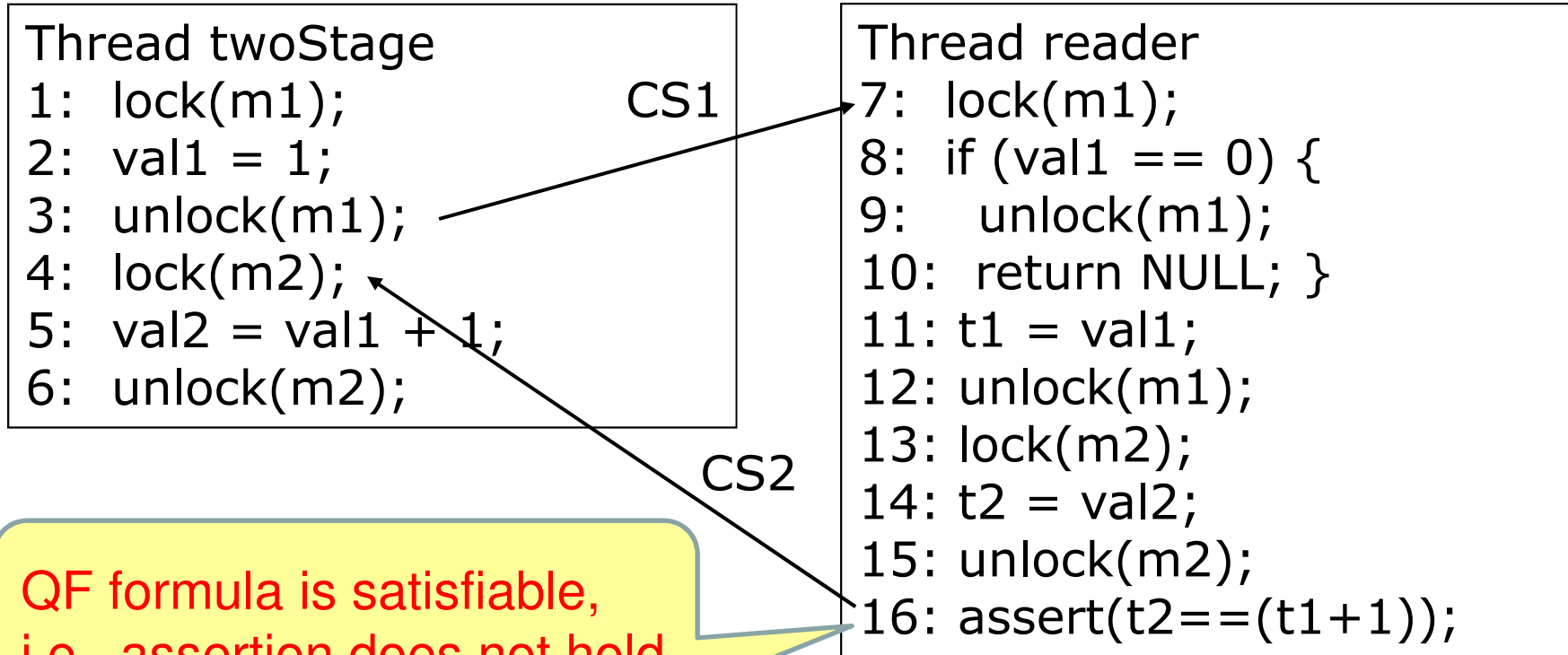
local variables: t1= 1; t2= 0;

# Lazy exploration: interleaving $I_f$

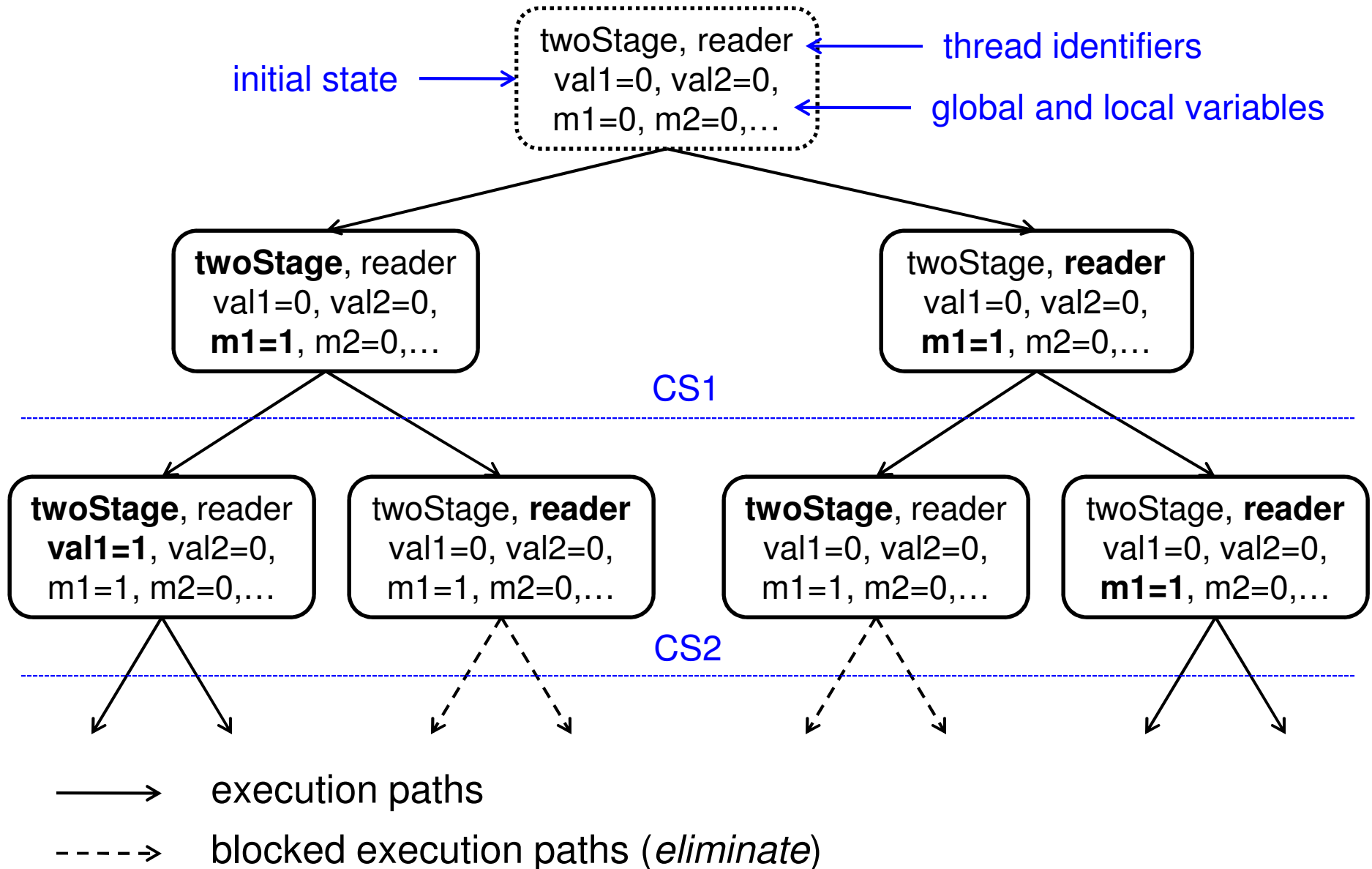
statements: 1-2-3-7-8-11-12-13-14-15-16-4-5-6

val1-access:  $W_{\text{twoStage},2}$  -  $R_{\text{reader},8}$  -  $R_{\text{reader},11}$  -  $R_{\text{twoStage},5}$

val2-access:  $R_{\text{reader},14}$  -  $W_{\text{twoStage},5}$



# Lazy Approach: State Transitions



# Observations about the lazy approach

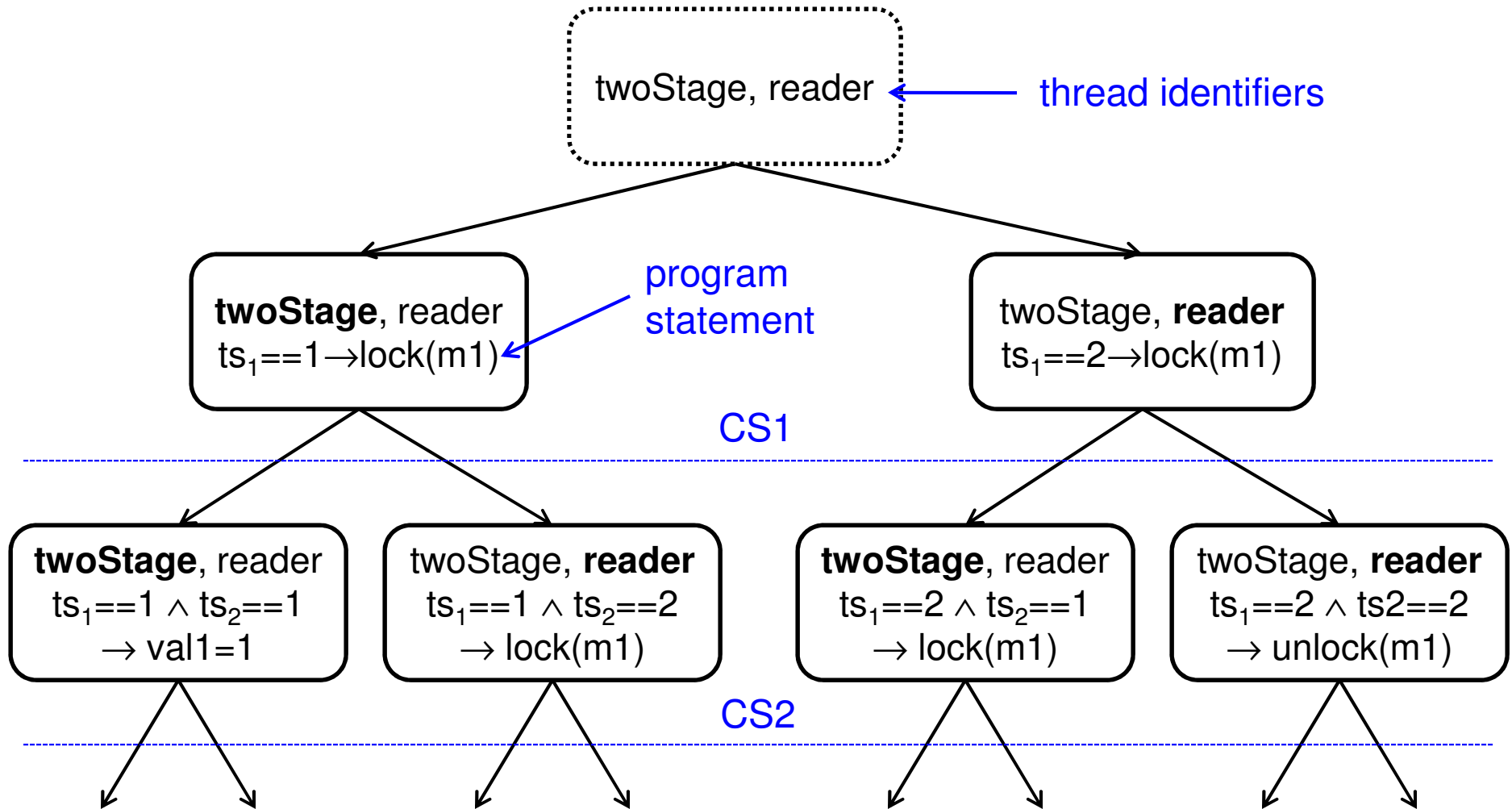
- naïve but useful:
  - bugs usually manifest with few context switches [Qadeer&Rehof'05]
  - keep in memory the parent nodes of all unexplored paths only
  - exploit which transitions are enabled in a given state
  - bound the number of preemptions ( $C$ ) allowed per threads
    - ▷ *number of executions:  $O(n^c)$*
  - as each formula corresponds to one possible path only, its size is relatively small
- can suffer performance degradation:
  - in particular for correct programs where we need to invoke the SMT solver once for each possible execution path

# Schedule Recording

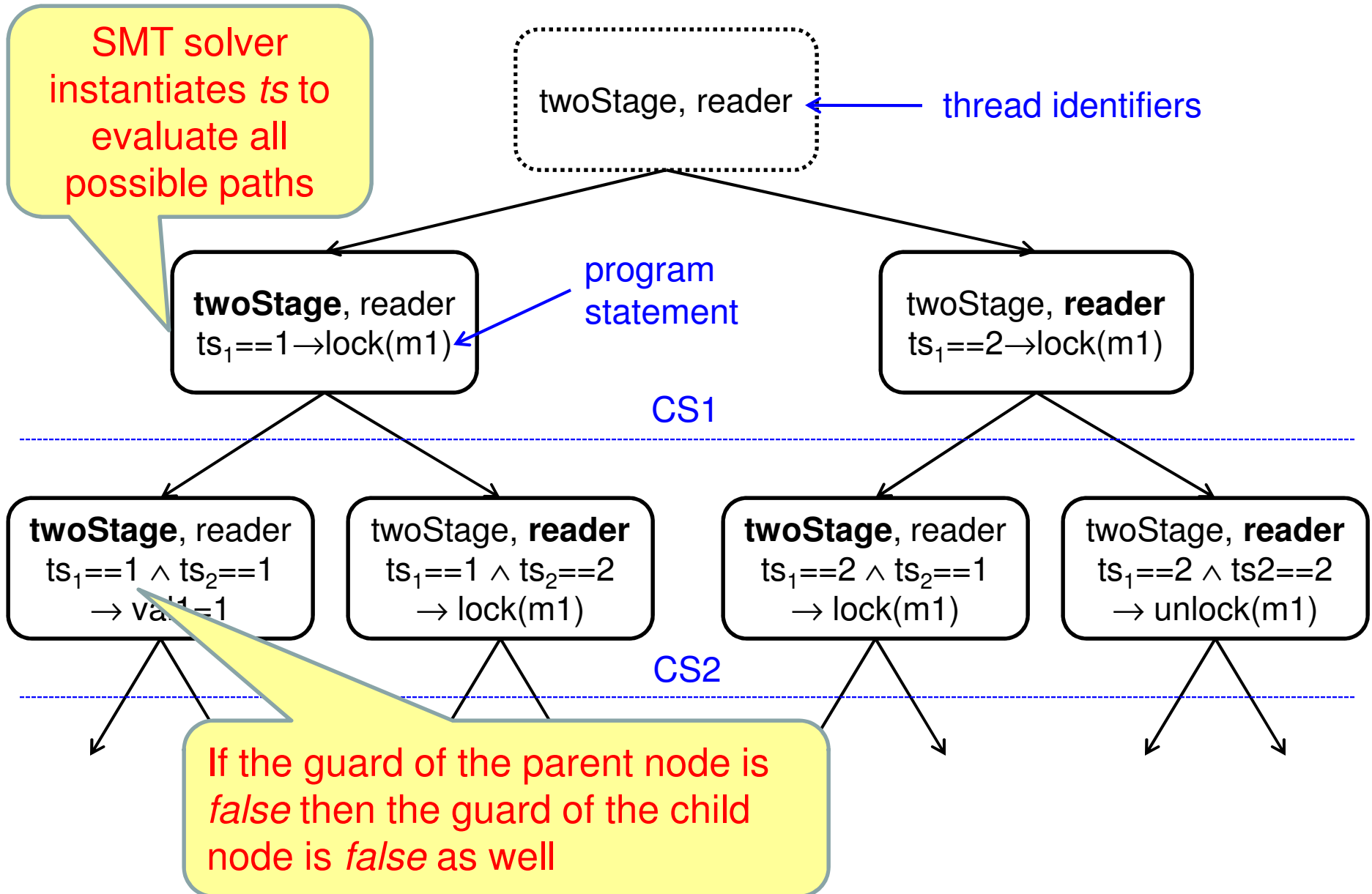
**Idea: systematically encode all possible interleavings into one formula**

- add a **fresh variable** ( $ts$ ) for each context switch block ( $i$ ) so that  $0 < ts_i \leq \text{number of threads}$ 
  - record in which order the *scheduler* has executed the program (*aka scheduler guards*)
  - SMT solver determines the order in which threads are simulated
- add scheduler guards only to **effective statements** (assignments and assertions)
  - record **effective context switches (ECS)**
    - ▷ *context switches to an effective statement*
  - *ECS block*: sequence of program statements that are executed with no intervening ECS

# Schedule Recording: Execution Paths



# Schedule Recording: Execution Paths



# Schedule Recording: Interleaving $I_s$

statements:

twoStage-ECS:

reader-ECS:

Thread twoStage

1: lock(m1);

2: val1 = 1;

3: unlock(m1);

4: lock(m2);

5: val2 = val1 + 1;

6: unlock(m2);

**ECS block:** sequence of program statements that are executed with no intervening ECS

10: return NULL; }

11: t1 = val1;

12: unlock(m1);

13: lock(m2);

14: t2 = val2;

15: unlock(m2);

16: assert(t2==(t1+1));



# Schedule Recording: Interleaving $I_s$

statements: 1

twoStage-ECS:  $ts_{1,1}$

reader-ECS:

guarded statement can only be executed if **statement 1** is scheduled in the **ECS block 1**

Thread twoStage

```

1: lock(m1);       $ts_1 == 1$ 
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);

```

Thread reader

```

7: lock(m1);
...
14: val2 = val1;
15: unlock(m2);
16: assert(t2==(t1+1));

```

each program statement is then prefixed by a *schedule guard*  $ts_i = j$ , where:

- $i$  is the **ECS block number**
- $j$  is the **thread identifier**

# Schedule Recording: Interleaving $I_s$

statements: 1-2

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$

reader-ECS:

Thread twoStage

1: lock(m1);  $ts_1 == 1$

● 2: val1 = 1;  $ts_2 == 1$

3: unlock(m1);

4: lock(m2);

5: val2 = val1 + 1;

6: unlock(m2);

Thread reader

7: lock(m1);

8: if (val1 == 0) {

9:   unlock(m1);

10: return NULL; }

11: t1 = val1;

12: unlock(m1);

13: lock(m2);

14: t2 = val2;

15: unlock(m2);

16: assert(t2==(t1+1));

# Schedule Recording: Interleaving $I_s$

statements: 1-2-3

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$

reader-ECS:

Thread twoStage

1: lock(m1);       $ts_1 == 1$

2: val1 = 1;       $ts_2 == 1$

● 3: unlock(m1);     $ts_3 == 1$

4: lock(m2);

5: val2 = val1 + 1;

6: unlock(m2);

Thread reader

7: lock(m1);

8: if (val1 == 0) {

9:    unlock(m1);

10: return NULL; }

11: t1 = val1;

12: unlock(m1);

13: lock(m2);

14: t2 = val2;

15: unlock(m2);

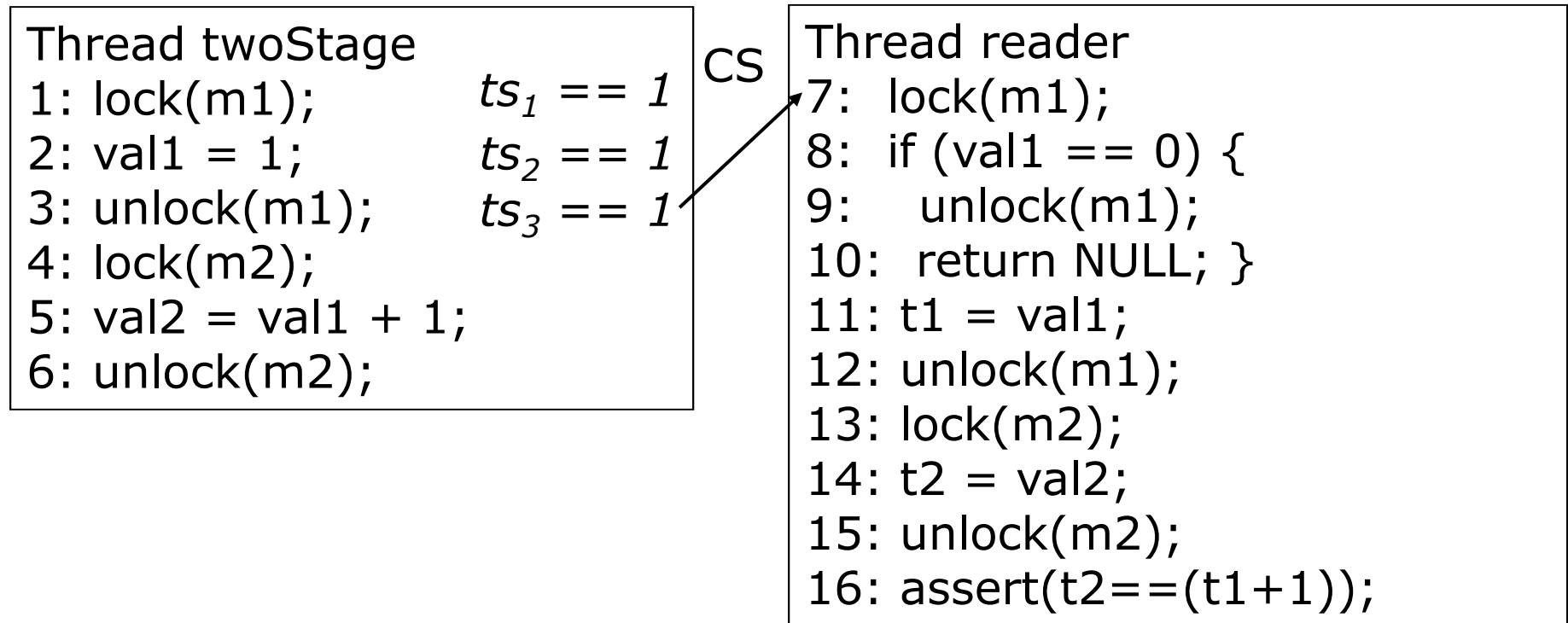
16: assert(t2==(t1+1));

# Schedule Recording: Interleaving $I_s$

statements: 1-2-3

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$

reader-ECS:

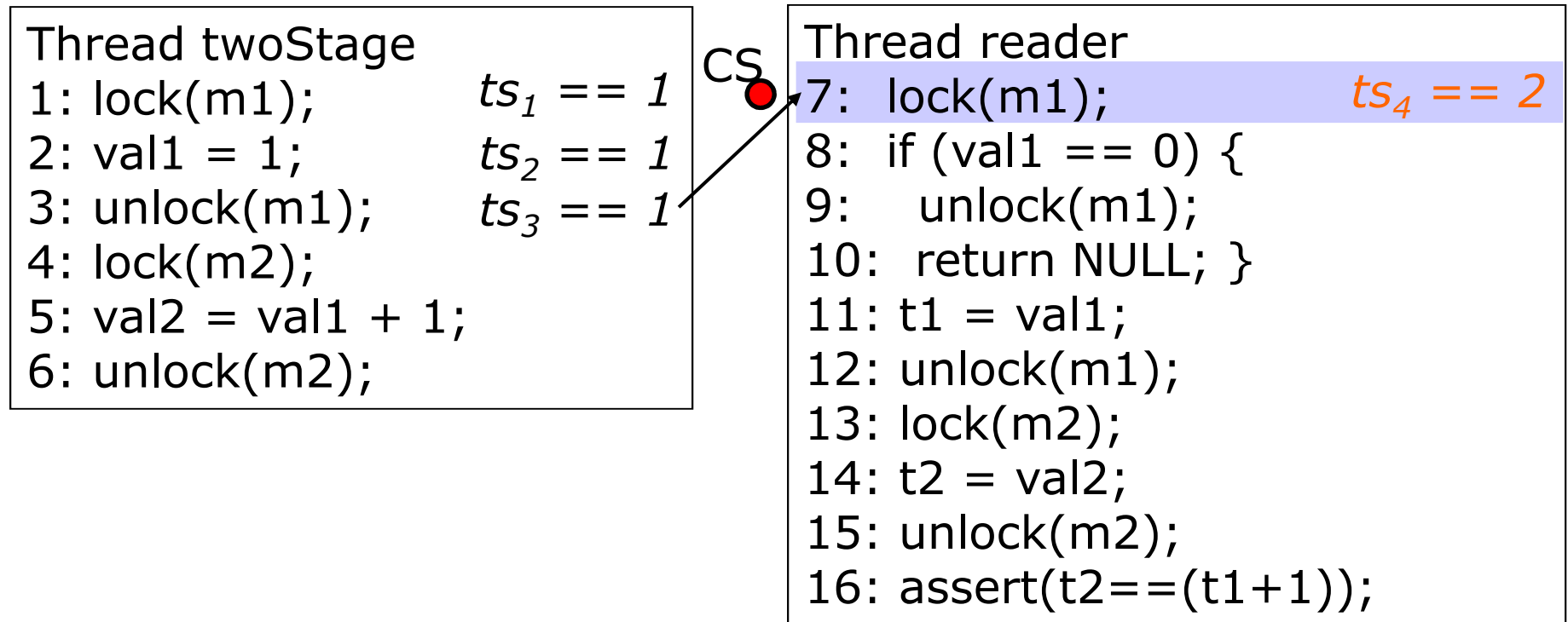


# Schedule Recording: Interleaving $I_s$

statements: 1-2-3-7

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$

reader-ECS:  $ts_{7,4}$

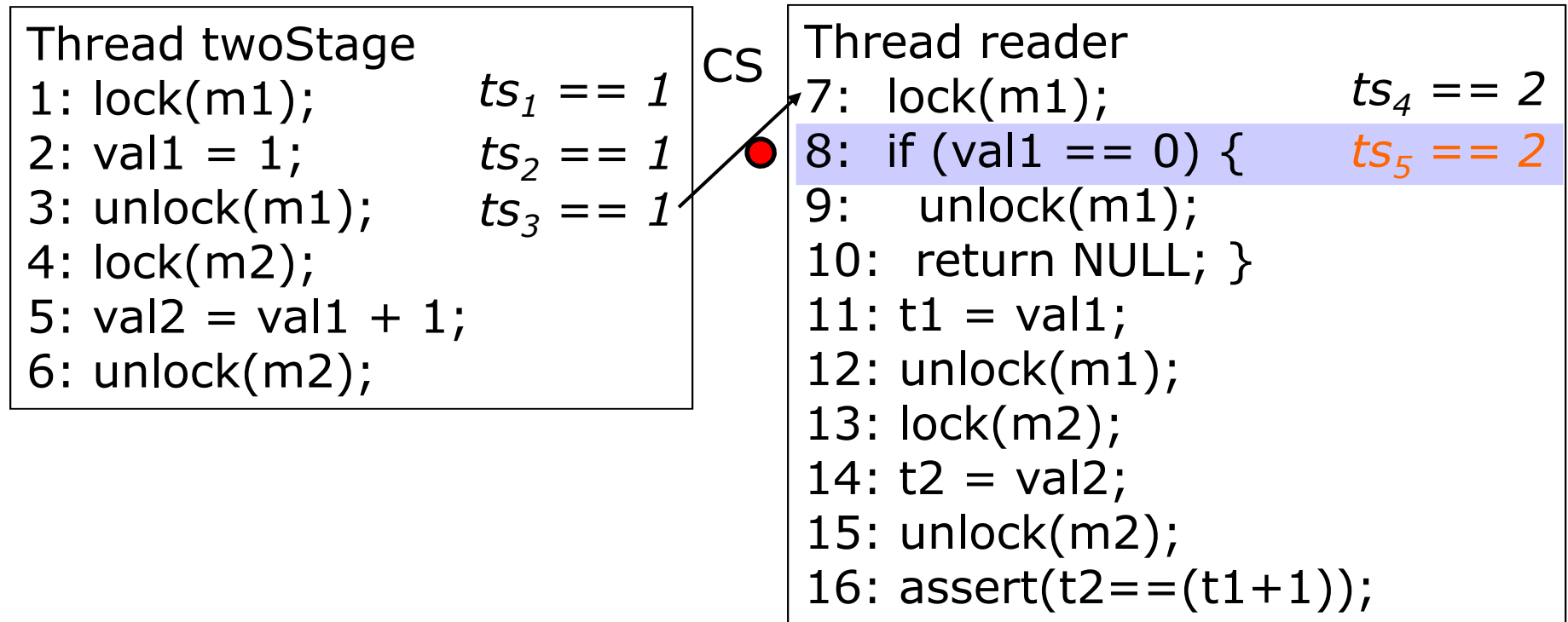


# Schedule Recording: Interleaving $I_s$

statements: 1-2-3-7-8

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$

reader-ECS:  $ts_{7,4}$ - $ts_{8,5}$

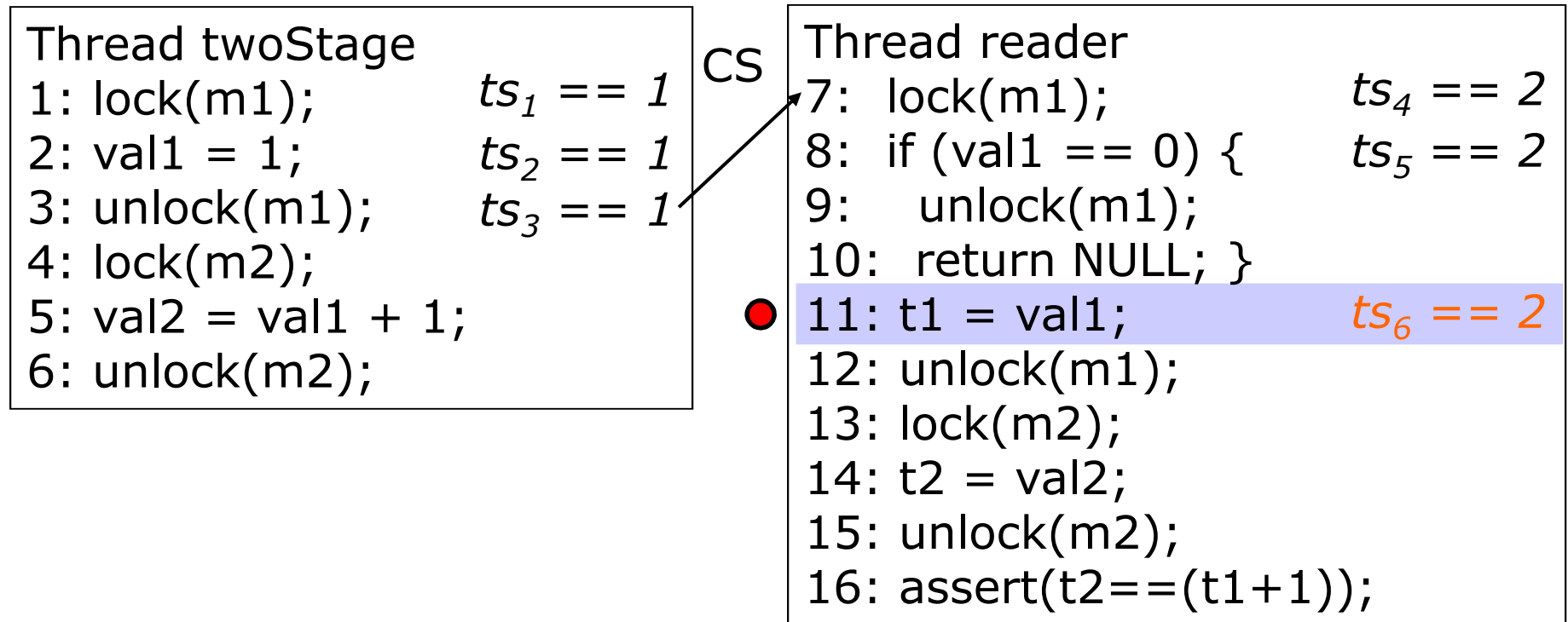


# Schedule Recording: Interleaving $I_s$

statements: 1-2-3-7-8-11

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$

reader-ECS:  $ts_{7,4}$ - $ts_{8,5}$ - $ts_{11,6}$

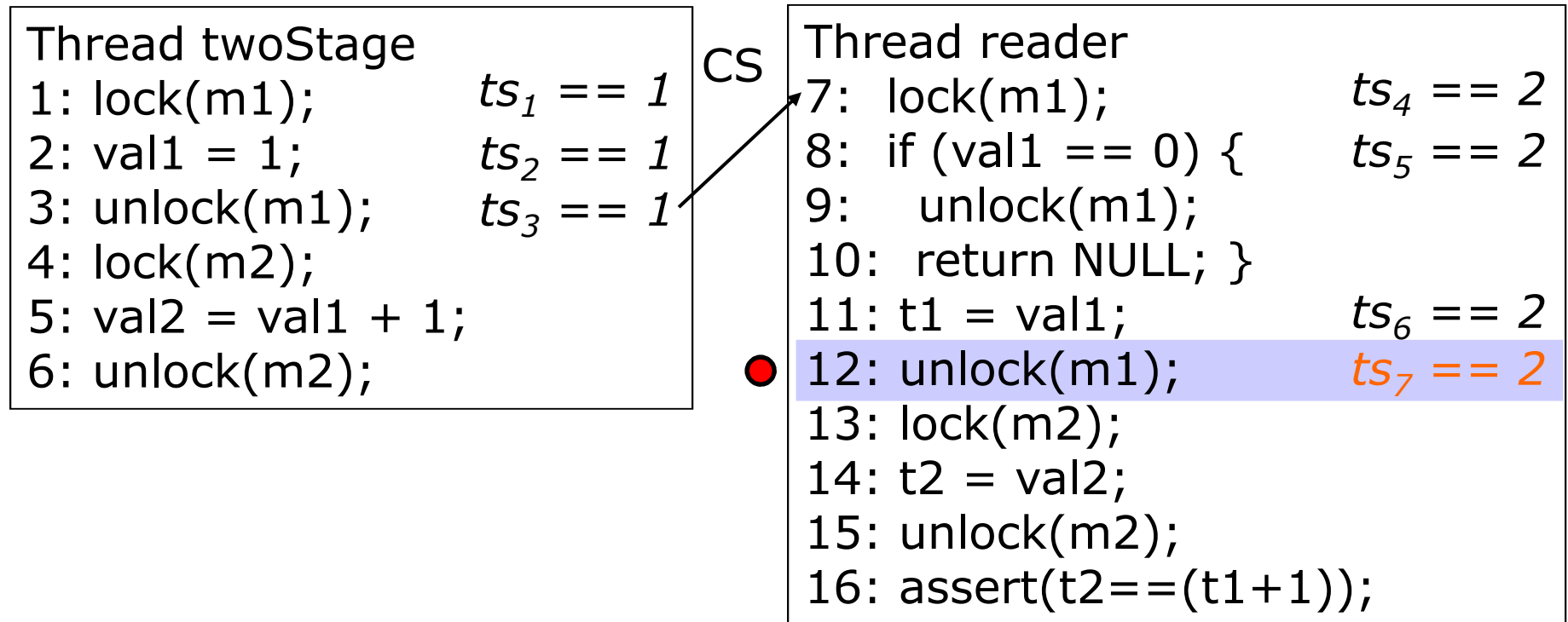


# Schedule Recording: Interleaving $I_s$

statements: 1-2-3-7-8-11-12

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$

reader-ECS:  $ts_{7,4}$ - $ts_{8,5}$ - $ts_{11,6}$ - $ts_{12,7}$



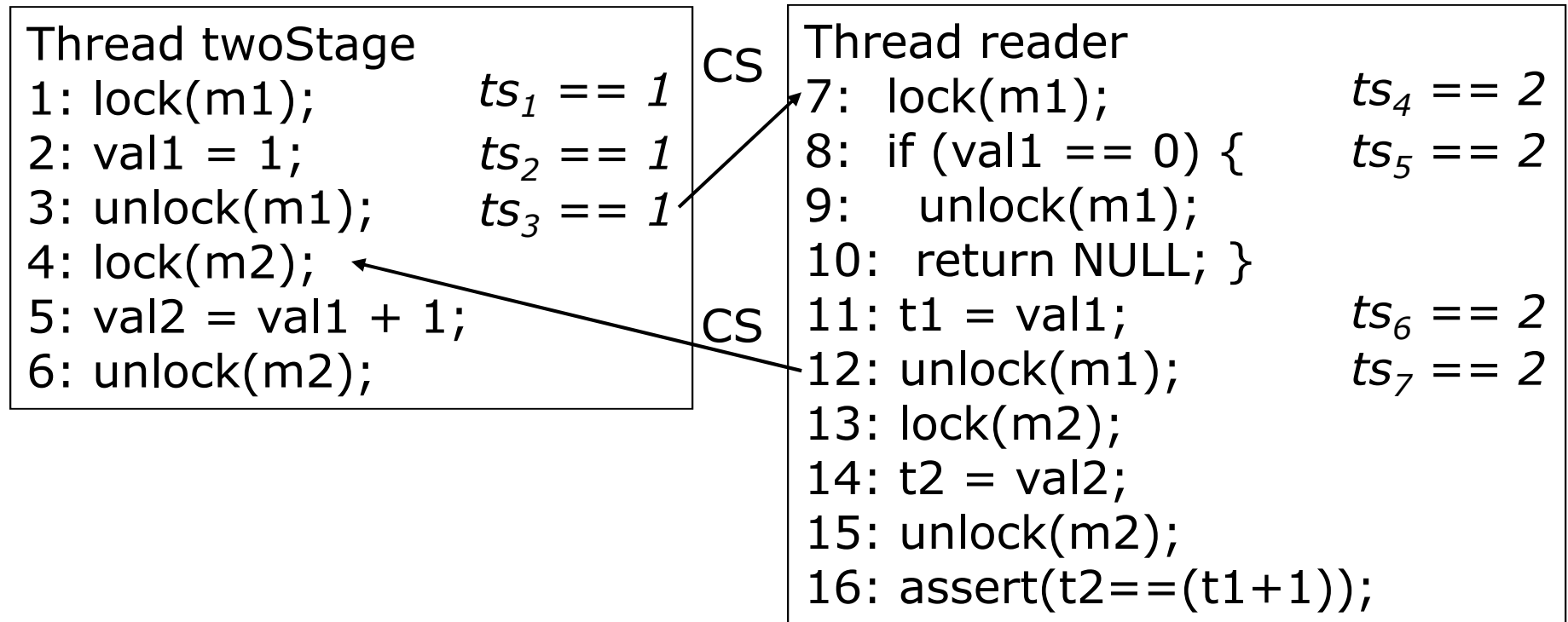


# Schedule Recording: Interleaving $I_s$

statements: 1-2-3-7-8-11-12

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$

reader-ECS:  $ts_{7,4}$ - $ts_{8,5}$ - $ts_{11,6}$ - $ts_{12,7}$

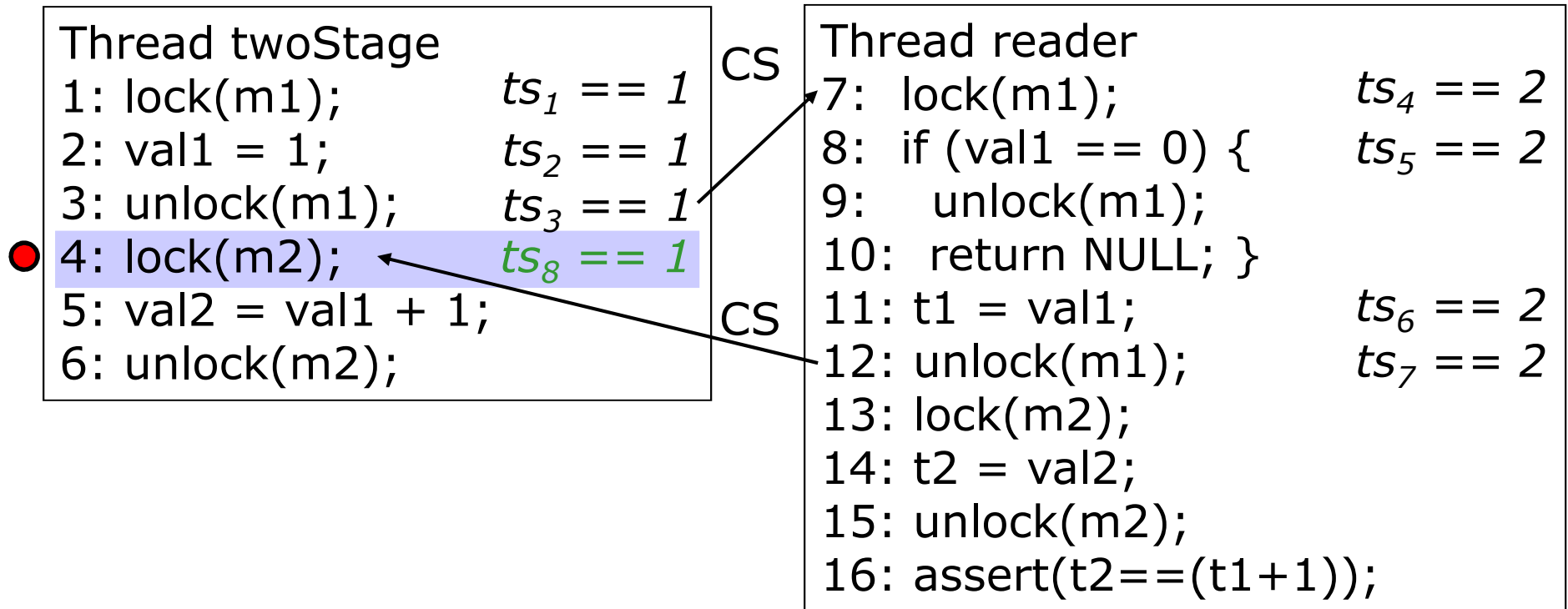


# Schedule Recording: Interleaving $I_s$

statements: 1-2-3-7-8-11-12-4

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$ - $ts_{4,8}$

reader-ECS:  $ts_{7,4}$ - $ts_{8,5}$ - $ts_{11,6}$ - $ts_{12,7}$

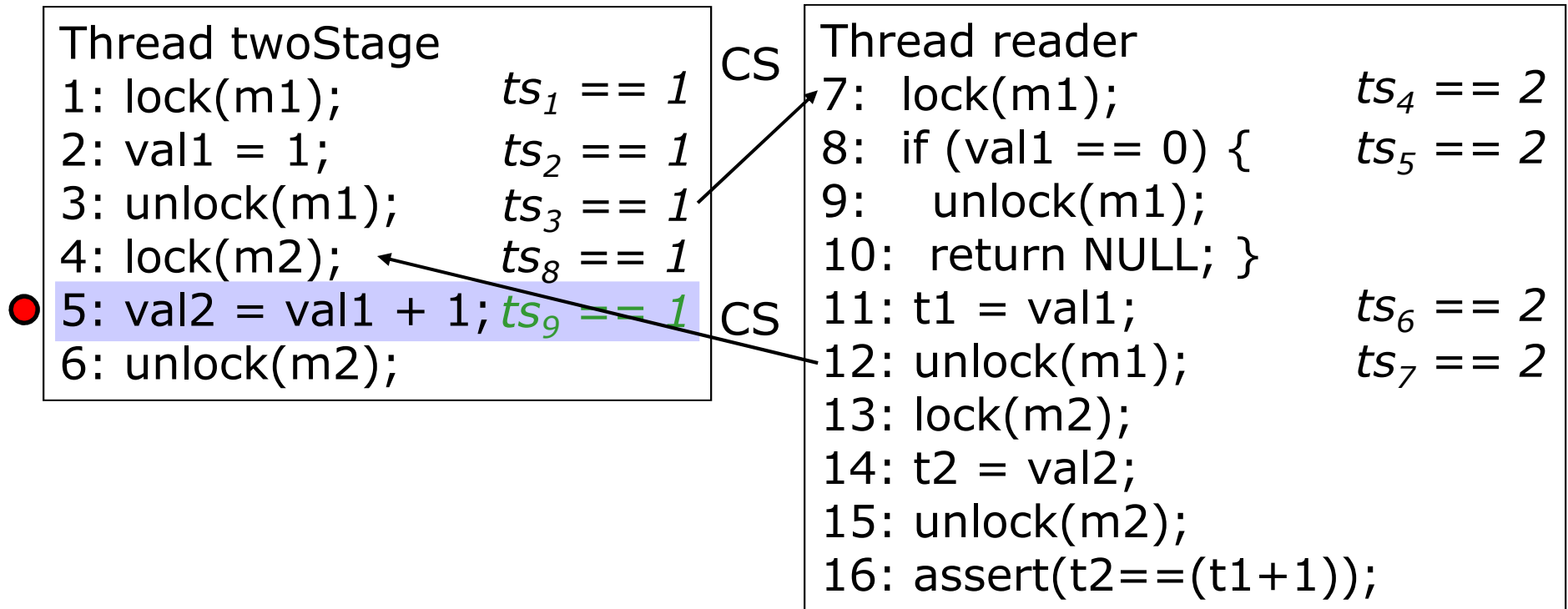


# Schedule Recording: Interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$ - $ts_{4,8}$ - $ts_{5,9}$

reader-ECS:  $ts_{7,4}$ - $ts_{8,5}$ - $ts_{11,6}$ - $ts_{12,7}$

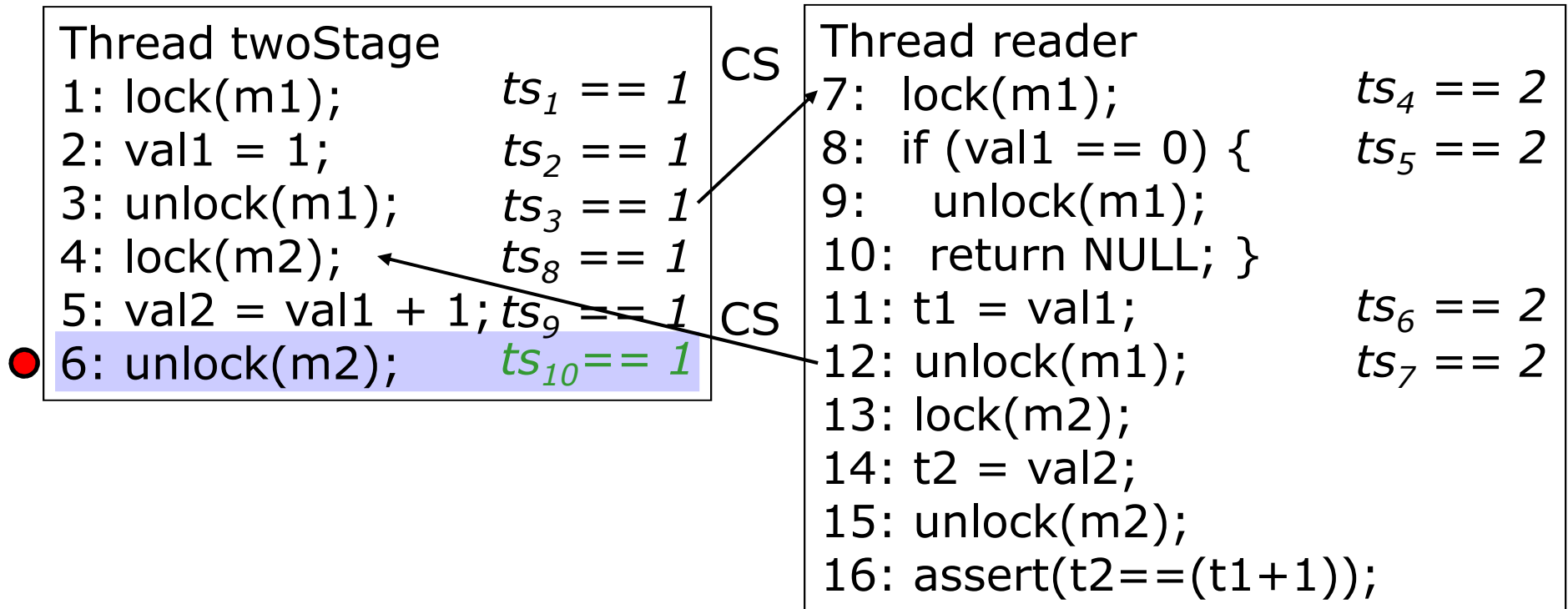


# Schedule Recording: Interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5-6

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$ - $ts_{4,8}$ - $ts_{5,9}$ - $ts_{6,10}$

reader-ECS:  $ts_{7,4}$ - $ts_{8,5}$ - $ts_{11,6}$ - $ts_{12,7}$

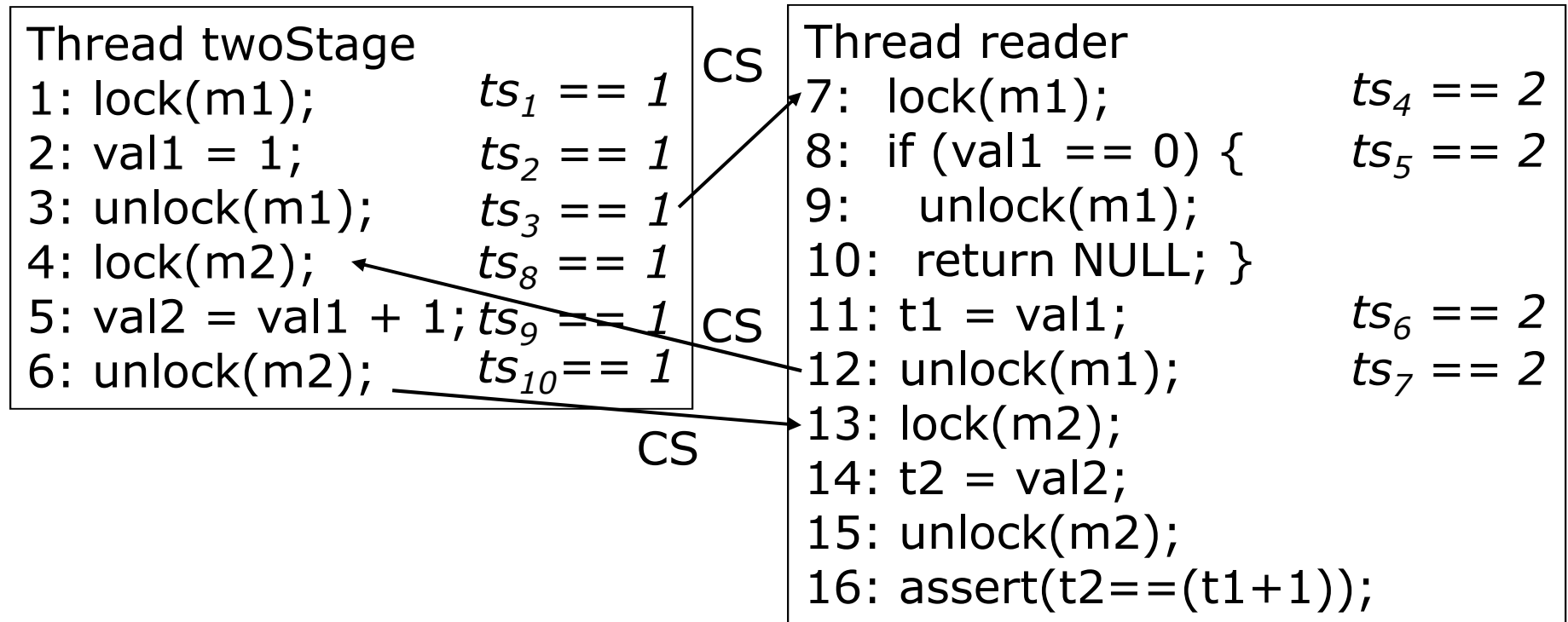


# Schedule Recording: Interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5-6

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$ - $ts_{4,8}$ - $ts_{5,9}$ - $ts_{6,10}$

reader-ECS:  $ts_{7,4}$ - $ts_{8,5}$ - $ts_{11,6}$ - $ts_{12,7}$

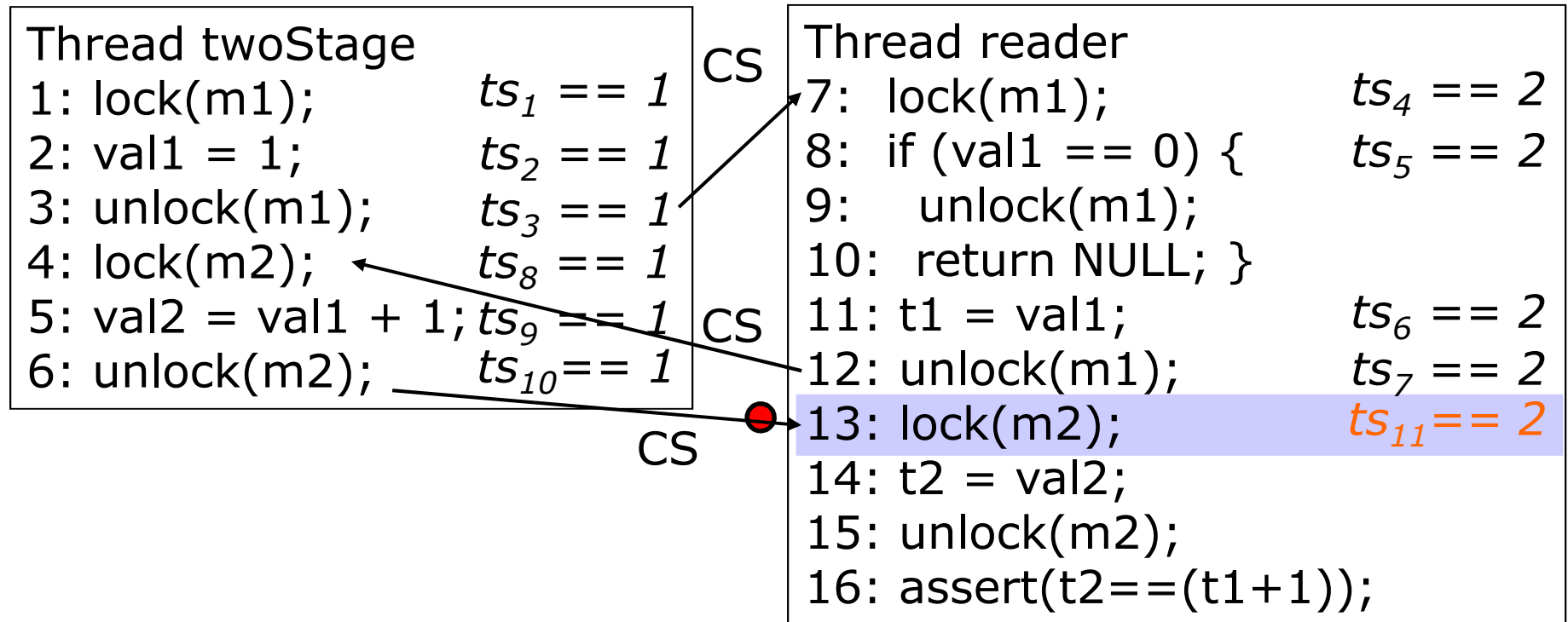


# Schedule Recording: Interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5-6-13

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$ - $ts_{4,8}$ - $ts_{5,9}$ - $ts_{6,10}$

reader-ECS:  $ts_{7,4}$ - $ts_{8,5}$ - $ts_{11,6}$ - $ts_{12,7}$ - $ts_{13,11}$

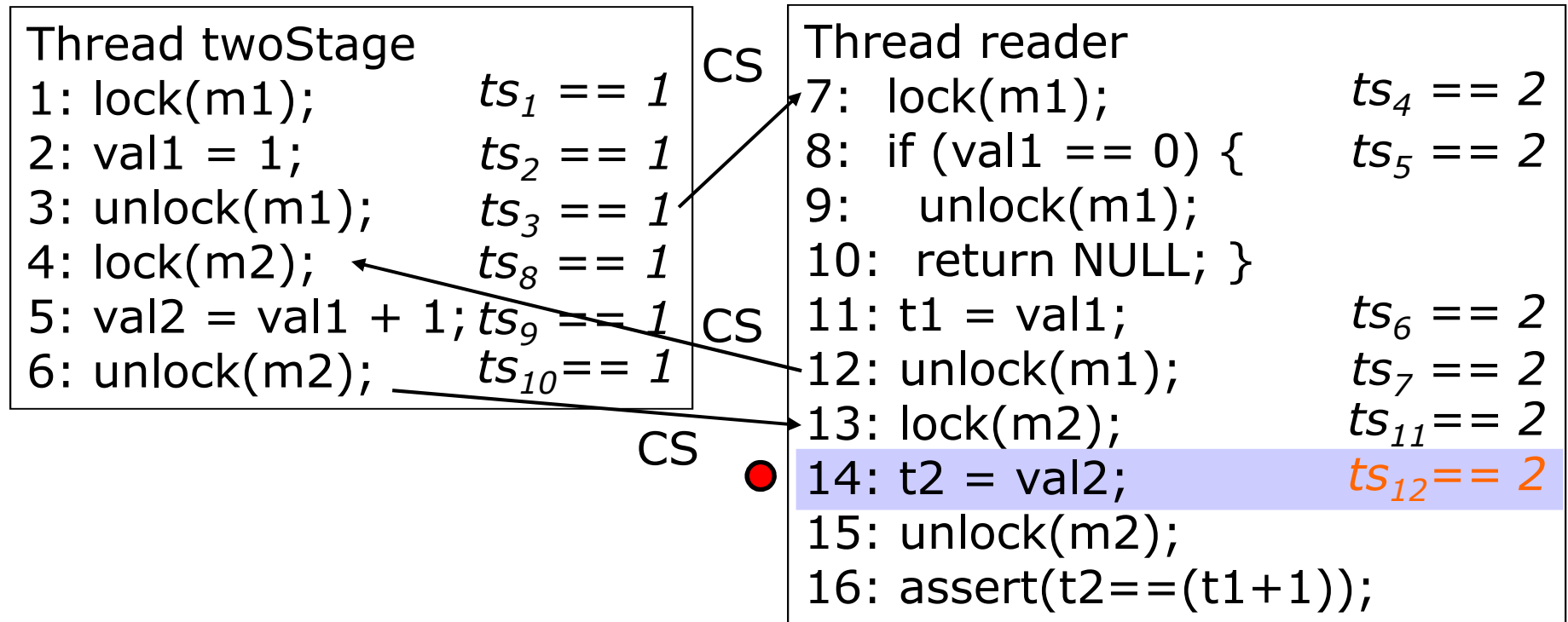


# Schedule Recording: Interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5-6-13-14

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$ - $ts_{4,8}$ - $ts_{5,9}$ - $ts_{6,10}$

reader-ECS:  $ts_{7,4}$ - $ts_{8,5}$ - $ts_{11,6}$ - $ts_{12,7}$ - $ts_{13,11}$ - $ts_{14,12}$

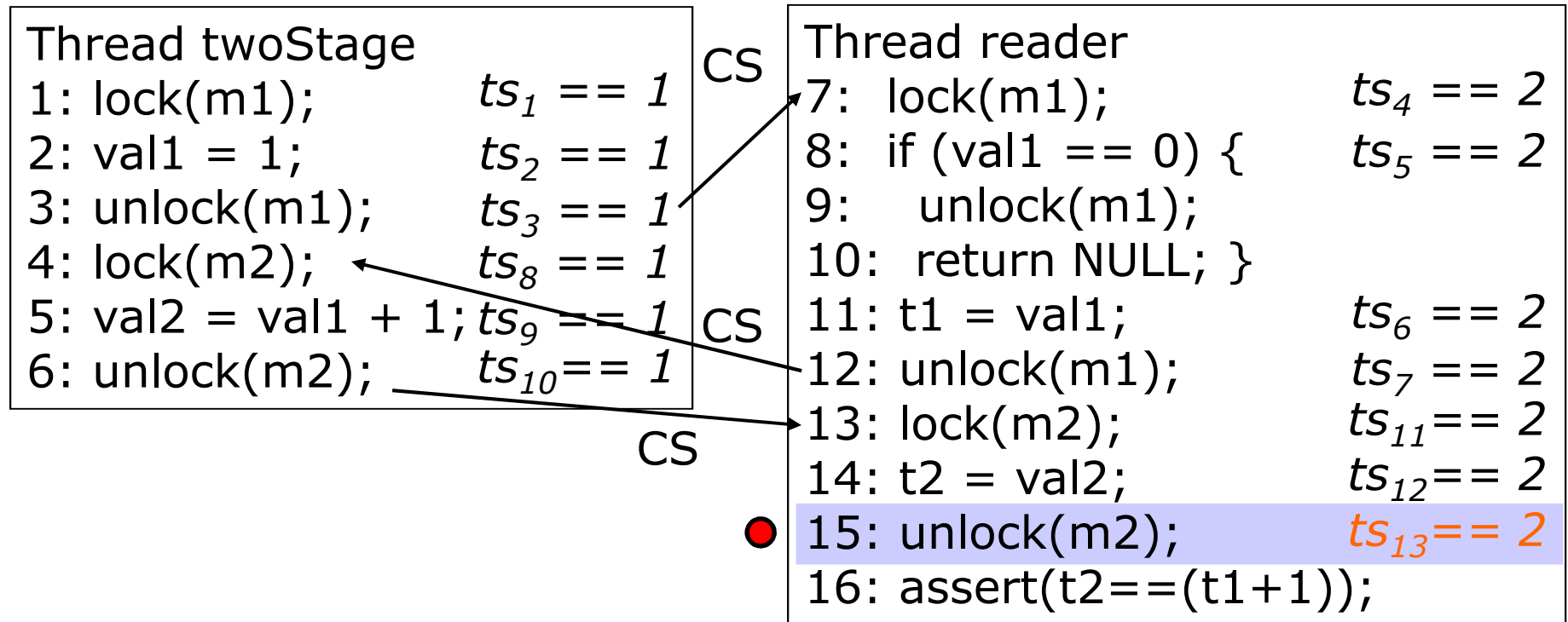


# Schedule Recording: Interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$ - $ts_{4,8}$ - $ts_{5,9}$ - $ts_{6,10}$

reader-ECS:  $ts_{7,4}$ - $ts_{8,5}$ - $ts_{11,6}$ - $ts_{12,7}$ - $ts_{13,11}$ - $ts_{14,12}$ - $ts_{15,13}$



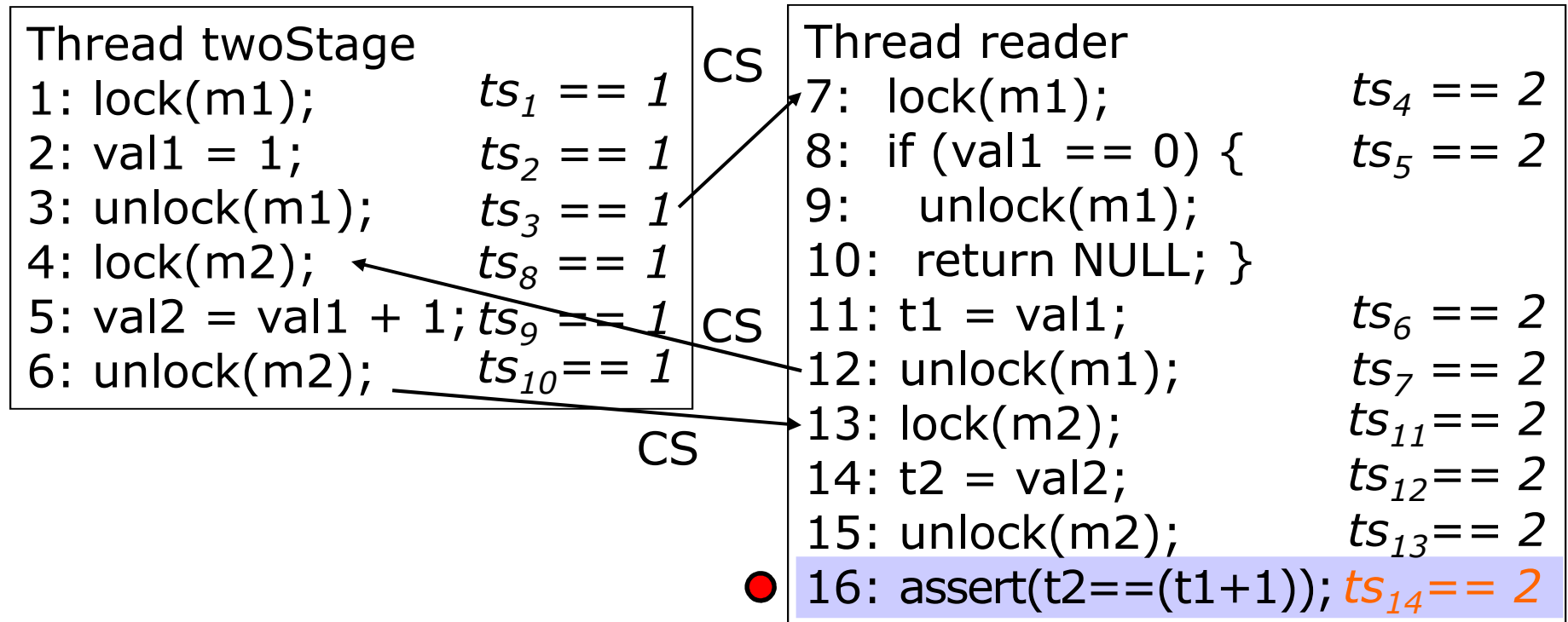


# Schedule Recording: Interleaving $I_s$

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15-16

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,2}$ - $ts_{3,3}$ - $ts_{4,8}$ - $ts_{5,9}$ - $ts_{6,10}$

reader-ECS:  $ts_{7,4}$ - $ts_{8,5}$ - $ts_{11,6}$ - $ts_{12,7}$ - $ts_{13,11}$ - $ts_{14,12}$ - $ts_{15,13}$ - $ts_{16,14}$

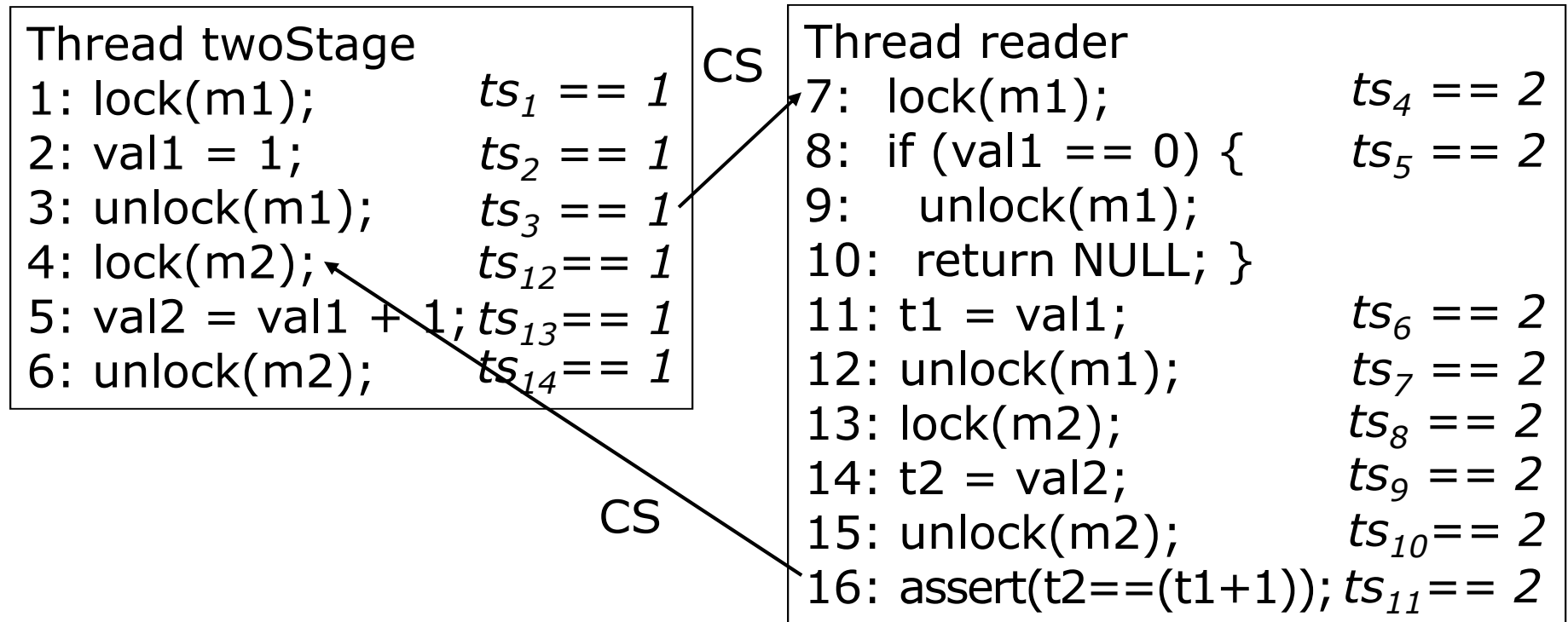


# Schedule Recording: Interleaving I<sub>f</sub>

statements: 1-2-3-7-8-11-12-13-14-15-16-4-5-6

twoStage-ECS:  $ts_{1,1}$ - $ts_{2,3}$ - $ts_{3,4}$ - $ts_{4,12}$ - $ts_{5,13}$ - $ts_{6,14}$

reader-ECS:  $ts_{7,4}$ - $ts_{8,5}$ - $ts_{11,6}$ - $ts_{12,7}$ - $ts_{13,8}$ - $ts_{14,9}$ - $ts_{15,10}$ - $ts_{16,11}$



# Observations about the schedule recoding approach

- we systematically explore the thread interleavings as before, but now:
  - add schedule guards to record in which order the scheduler has executed the program
  - encode all execution paths into one formula
    - ▷ *bound the number of preemptions*
    - ▷ *exploit which transitions are enabled in a given state*
- the number of threads and context switches can grow very large quickly, and easily “blow-up” the solver:
  - there is a clear trade-off between usage of time and memory resources

# Under-approximation and Widening

**Idea: check models with an increased set of allowed interleavings [Grumberg&et al.'05]**

- start from a single interleaving (under-approximation) and widen the model by adding more interleavings incrementally
- main steps of the algorithm:
  1. encode control literals ( $cl_{i,j}$ ) into the verification condition  $\psi$ 
    - ▷  $cl_{i,j}$  where  $i$  is the ECS block number and  $j$  is the thread identifier
  2. check the satisfiability of  $\psi$  (stop if  $\psi$  is satisfiable)
  3. extract proof objects generated by the SMT solver
  4. check whether the proof depends on the control literals (stop if the proof does not depend on the control literals)
  5. remove literals that participated in the proof and go to step 2

# UW Approach: Running Example

- use the same guards as in the schedule recording approach as control literals
  - but here the schedule is updated based on the information extracted from the proof

Thread twoStage	
1: lock(m1);	$cl_{1,twoStage} \rightarrow ts_1 == 1$
2: val1 = 1;	$cl_{2,twoStage} \rightarrow ts_2 == 1$
3: unlock(m1);	$cl_{3,twoStage} \rightarrow ts_3 == 1$
4: lock(m2);	$cl_{8,twoStage} \rightarrow ts_8 == 1$
5: val2 = val1 + 1;	$cl_{9,twoStage} \rightarrow ts_9 == 1$
6: unlock(m2);	$cl_{10,twoStage} \rightarrow ts_{10} == 1$

- reduce the number of control points from  $m \times n$  to  $e \times n$ 
  - $m$  is the number of program statements;  $n$  is the number of threads, and  $e$  is the number of ECS blocks

# Evaluation

# Comparison of the Approaches

- Goal: compare efficiency of the proposed approaches
  - lazy exploration
  - schedule recording
  - under-approximation and widening
- Set-up:
  - ESBMC v1.6 together with the SMT solver Z3 v2.7
  - support the logics *QF\_AUFBV* and *QF\_AUFLIRA*
  - standard desktop PC, time-out 3600 seconds

# About the benchmarks

	Module	#L	#P	k	#T	#C	
		81	47	17	26	2	Franklin file system with array
2	fsbench_bad		48		27	2	Franklin file system with array
3	in		1	1	13	4	inconsistency to a hash table
4	aget-0.4_bad	1233	1				aged download or
5	reorder_bad	84					a data race
6	twostage_bad	128	8	16	30	3	contains an atomicity violation
7	wronglock_bad	110	8	5	8	7	threads do not obtain the same lock instance
8	exStbHDMI_ok	1060	25	16	2	20	set various capabilities in the HDMI device
9	exStbLED_ok	425	45	10	2	-	front panel LED display
10	exStbThumb_bad	1109	243	2	2	1	Demonstrate how thumbnail images can be displayed and layer blending can be used

*lines of code*

*number of context switches*

*number of properties checked*

*Number of threads*

*the number of BMC unrolling steps*



# About the benchmarks

*Inspect  
 benchmark  
 suite*

	Module						Description
1	fsbench_ok						Frangipani file system
2	fsbench_bad		48	28	27	2	Frangipani file system with array out of bounds
3	indexer_ok	77	21	129	13	4	insert messages into a hash table concurrently
4	aget-0.4_bad	1233	170	257	3	2	Multi-threaded download accelerator
5	reorder_bad	84	16	11	20	3	contains a data race
6	twostage_bad	128	8	16	30	3	contains an atomicity violation
7	wronglock_bad	110	8	5	8	7	threads do not obtain the same lock instance
8	exStbHDMI_ok	1060	25	16	2	20	set various capabilities in the HDMI device
9	exStbLED_ok	425	45	10	2	-	front panel LED display
10	exStbThumb_bad	1109	243	2	2	1	Demonstrate how thumbnail images can be displayed and layer blending can be used

# About the benchmarks

	Module	#L	#P	k	#T	#C	Description
1	fsbench_ok	81	47	27	26	2	Frangipani file system
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3	indexer_ok					4	insert messages into a hash table concurrently
4	aget-0.4_bad	12	170	257	3	2	Multi-threaded download accelerator
5	reorder_bad	84	16	11	20	3	contains a data race
6	twostage_bad	128	8	16	30	3	contains an atomicity violation
7	wronglock_bad	110	8	5	8	7	threads do not obtain the same lock instance
8	exStbHDMI_ok	1060	25	16	2	20	set various capabilities in the HDMI device
9	exStbLED_ok	425	45	10	2	-	front panel LED display
10	exStbThumb_bad	1109	243	2	2	1	Demonstrate how thumbnail images can be displayed and layer blending can be used

*VV-lab  
benchmark  
suite*

# About the benchmarks

	Module	#L	#P	k	#T	#C	Description
1	fsbench_ok	81	47	27	26	2	Frangipani file system
2	fsbench_bad	80	48	28	27	2	Frangipani file system with array out of bounds
3	indexer_ok	77	21	129	13	4	insert messages into a hash table concurrently
4	aget-0.4_bad						Multi-threaded download accelerator
5	reorder_bad						contains a data race
6	twostage_bad						contains an atomicity violation
7	wronglock_bad		8	5	8	7	threads do not obtain the same lock instance
8	exStbHDMI_ok	1060	25	16	2	20	set various capabilities in the HDMI device
9	exStbLED_ok	425	45	10	2	-	front panel LED display
10	exStbThumb_bad	1109	243	2	2	1	Demonstrate how thumbnail images can be displayed and layer blending can be used

*Set-top box applications from NXP semiconductors*

# Comparison of the approaches (1)

Module	Lazy					UW						
	Time	#P		#I	#I/ #IF	Total	Fail	Error	Total	#P		Iter
		Fail	Error	Fail						Error		
fsben	729	1	0	1/0	729	1	0	1770	1	0	2	
fsben	<1	0	0	0/0	<1	0	0	0	0	0	1	
index	0	0	0	0/0	0	0	0	234	0	0	1	
aget-0.4_bad	248	1	0	2/1	254	1	0	245	1	0	1	
reorder_bad	<1	1	0	32/20	<1	1	0	2	1	0	3	
twostage_bad	3	1	0	41/15	2	1	0	7	1	0	4	
wronglock_bad	84	1	0	20507/ 16	293	1	0	2089	1	0	3	
exStbHDMI_ok	268	0	0	1/0	260	0	0	278	0	0	1	
exStbLED_ok	900	0	0	11/0	MO	0	1	MO	0	1	1	
exStbThumb_bad	14	1	0	3/1	MO	0	1	MO	0	1	1	

encoding and  
solver time

number of  
generated and  
failed interleavings

error detected  
in module –  
**GOOD THING**

error occurred  
in tool –  
**BAD THING**

number of  
iterations

# Comparison of the approaches (1)

*lazy encoding often more efficient than schedule recording and UW*

Name	Lazy			Schedule			UW			
	#P	#I	#I/ #IF	Time	#P		Time	#P		Iter
	Error			Total	Fail	Error	Total	Fail	Error	
fsbench_bad	1	1	729/729	452	1	0	1779	1	0	2
fsbench_ok	282	0	676/0	334	0	0	335	0	0	1
indexer_ok	575	0	17160/0	238	0	0	234	0	0	1
aget-0.4_bad	248	1	2/1	254	1	0	245	1	0	1
reorder_bad	<1	1	32/20	<1	1	0	2	1	0	3
twostage_bad	3	1	41/15	2	1	0	7	1	0	4
wronglock_bad	84	1	20507/16	293	1	0	2089	1	0	3
exStbHDMI_ok	268	0	1/0	260	0	0	278	0	0	1
exStbLED_ok	900	0	11/0	MO	0	1	MO	0	1	1
exStbThumb_bad	14	1	3/1	MO	0	1	MO	0	1	1

# Comparison of the approaches (2)

*lazy encoding often more efficient than schedule recording and UW, but not always*

M				Schedule				UW			
				#I	Time	#P		Time	#P		
				#I/ #IF	Total	Fail	Error	Total	Fail	Error	Iter
fsbench_bad		1	0	729/ 729	452	1	0	1779	1	0	2
fsbench_ok	32	0	0	676/0	334	0	0	335	0	0	1
indexer_ok	575	0	0	17160/ 0	238	0	0	234	0	0	1
aget-0.4_bad	248	1	0	2/1	254	1	0	245	1	0	1
reorder_bad	<1	1	0	32/20	<1	1	0	2	1	0	3
twostage_bad	3	1	0	41/15	2	1	0	7	1	0	4
wronglock_bad	84	1	0	20507/ 16	293	1	0	2089	1	0	3
exStbHDMI_ok	268	0	0	1/0	260	0	0	278	0	0	1
exStbLED_ok	900	0	0	11/0	MO	0	1	MO	0	1	1
exStbThumb_bad	14	1	0	3/1	MO	0	1	MO	0	1	1

# the approaches (3)

*lazy encoding is extremely fast for satisfiable instances*

Module	Lazy				Schedule			UW			
	Total	Fail	Error	#I/ #IF	Total	Fail	Error	Total	Fail	Error	Iter
fsbench_bad	1	1	0	729/ 729	452	1	0	1779	1	0	2
fsbench_ok	282	0	0	676/0	334	0	0	335	0	0	1
indexer_ok	575	0	0	17160/ 0	238	0	0	234	0	0	1
aget-0.4_bad	248	1	0	2/1	254	1	0	245	1	0	1
reorder_bad	<1	1	0	32/20	<1	1	0	2	1	0	3
twostage_bad	3	1	0	41/15	2	1	0	7	1	0	4
wronglock_bad	84	1	0	20507/ 16	293	1	0	2089	1	0	3
exStbHDMI_ok	268	0	0	1/0	260	0	0	278	0	0	1
exStbLED_ok	900	0	0	11/0	MO	0	1	MO	0	1	1
exStbThumb_bad	14	1	0	3/1	MO	0	1	MO	0	1	1

# Results

- **lazy, schedule recording, and UW** algorithms
  - *lazy*: check constraints lazily is *fast for satisfiable instances*
  - *schedule recording*: the number of threads and context switches can grow quickly (and easily “blow-up” the model checker)
  - *UW*: memory overhead and slowdowns to extract the *unsat core*
- handle more than two threads, detect concurrency bugs (e.g., atomicity and order violations, deadlocks)
- [users.ecs.soton.ac.uk/lcc08r/esbmc/](http://users.ecs.soton.ac.uk/lcc08r/esbmc/)

# Future Work

- partial order reduction (static and dynamic)
- interpolants to prove no interference of context switches
- fault localization in multi-threaded C programs