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Assisted Counterexample-Guided Inductive Optimization for Robot Path Planning

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Synopsis

- Background knowledge
- Objectives
- ACEGIO-based path planning algorithm
- Experimental Evaluation
- Conclusion and Future work

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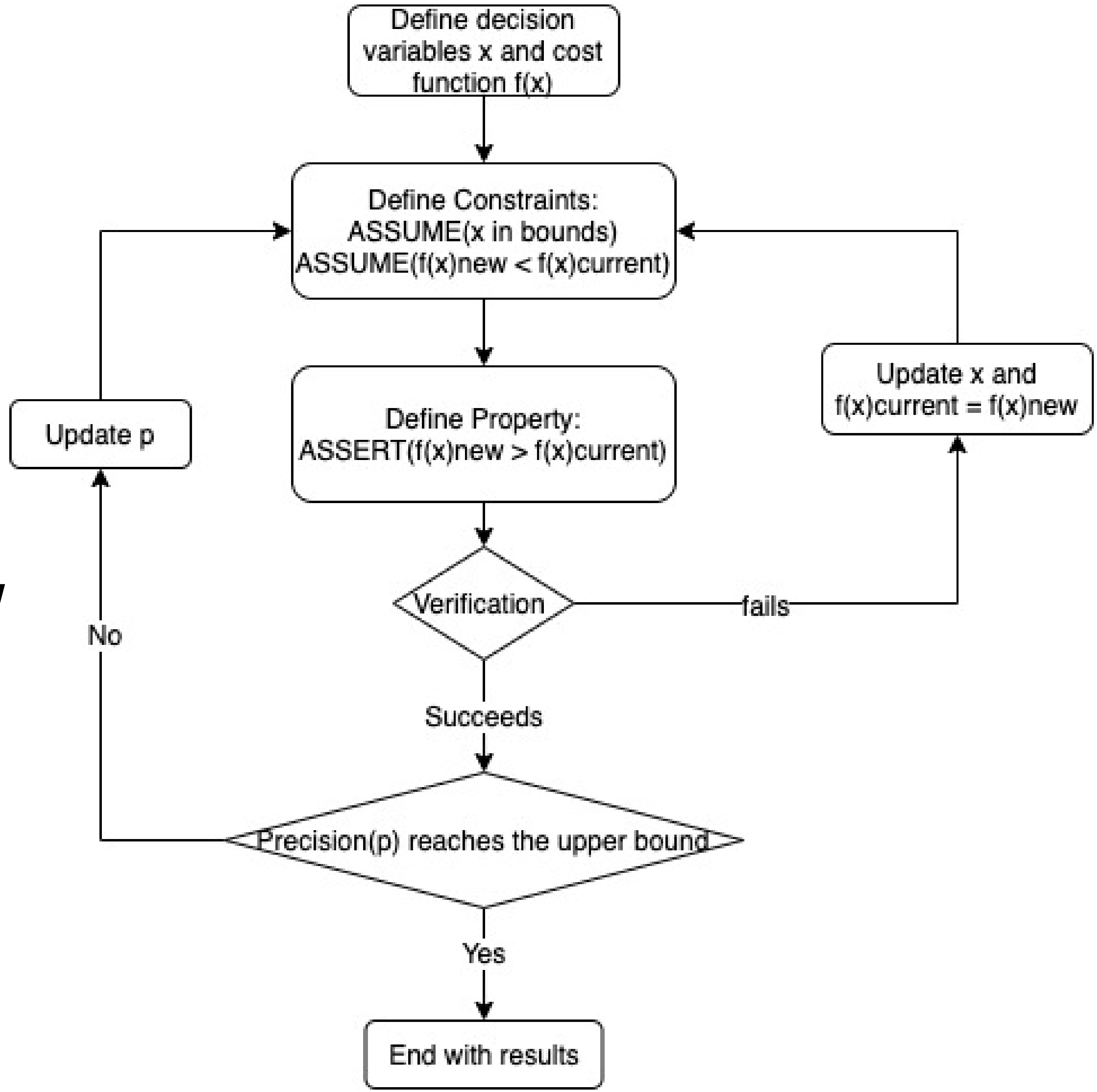
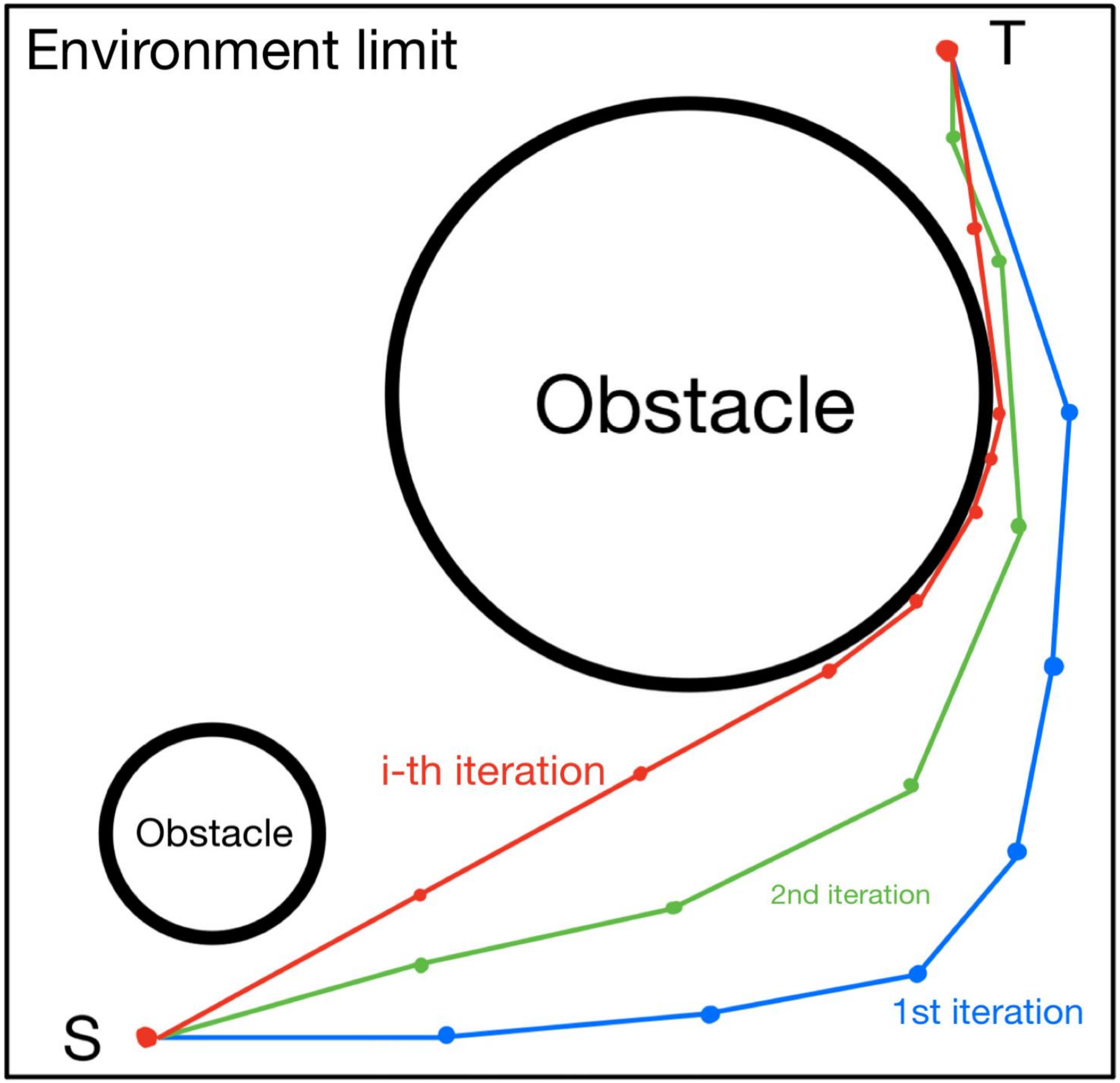
CEGIO work flow



Background knowledge



- Path planning can be considered as an optimization problem
- CEGIO can be applied to achieve the optimal path by iteratively requesting counterexamples from SAT/SMT

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Background knowledge

- However, requesting counterexamples from SAT/SMT solvers is the most time-consuming process
- ACEGIO combines CEGIO with an auxiliary algorithm (Gradient Descent is selected in this work)
- ACEGIO relies on CEGIO to preserve the optimization ability and relies on the selected auxiliary algorithm to improve the efficiency

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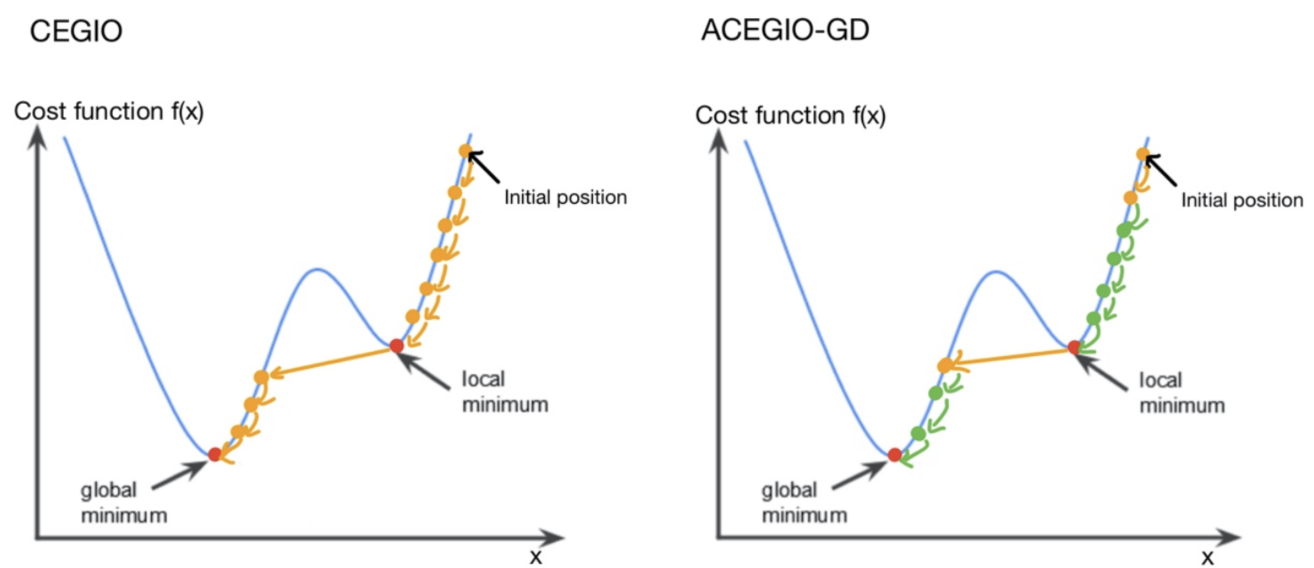
Illustrative Example

The optimization process of employing ACEGIO-GD contains fewer times of requesting counterexamples from SAT/SMT solvers, and Gradient Descent (much faster) is applied instead.



- The red dot is global minimum or local minimum
- The orange dot is position updated by requesting counterexamples from SMT solvers
- The green dot is position updated by Gradient Descent
- The orange arrow is the optimizing path generated by requesting counterexamples
- The green arrow is the optimizing path generated by Gradient Descent

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Objectives

The main objective of this work is to propose and evaluate a novel offline mobile robot path planning algorithm based on Assisted Counterexample-Guided Inductive Optimization

- Develop ACEGIO algorithm with Gradient Descent as the auxiliary algorithm to generate optimal paths
- Evaluate the proposed ACEGIO-based path planning algorithm
- Compare the results with other traditional optimization techniques based path planning algorithms

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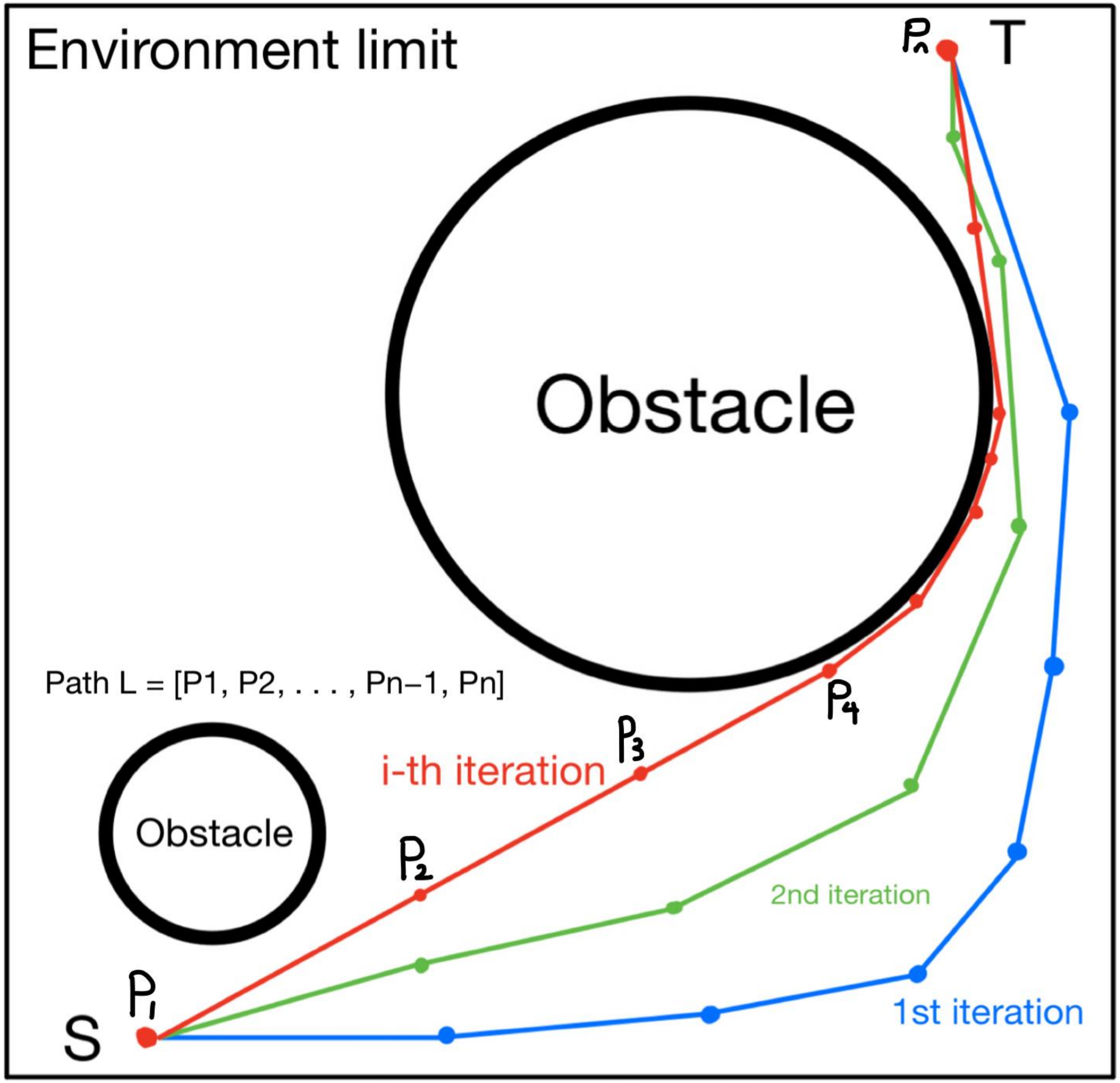
Two steps of applying ACEGIO to path planning problems

1. Formulate the path planning problem as an optimization problem (define decision variables, cost function and set constraints)
2. Apply ACEGIO-GD to find the optimal path that minimizes the cost function and satisfies the constraints

Path planning problem formulation



- Starting position S(P1) and target position T(Pn)
- Points consisted the path L = [P1, P2, . . . , Pn−1, Pn]

Path planning problem formulation

- The cost function: $J(\mathrm{L}) = \sum_{\mathrm{i}=1}^{\mathrm{n}-1} \|\mathrm{P}_{\mathrm{i}+1} - \mathrm{P}_{\mathrm{i}}\|_2$, n is the number of points on the desired path
- Obstacles $\mathbb{O}$ and environments limits $\mathbb{E}$
- Optimization problem for path planning can be written as

$$\begin{aligned} \min_L \quad & \mathrm{J}(\mathrm{L}) \\ \text{s.t.} \quad & \mathrm{p}_{\mathrm{i}\lambda}(\mathrm{L}) \notin \mathbb{O} \\ & \mathrm{p}_{\mathrm{i}\lambda}(\mathrm{L}) \in \mathbb{E} \\ & \mathrm{i} = 1, \ldots, \mathrm{n}-1 \end{aligned}$$

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CEGIO-based path planning algorithm

input : Cost function $J(\mathbf{L})$, is a set of obstacles constraints $\mathbb{O}$ and a set of environment constraints $\mathbb{E}$, which define Ω and a desired precision η

output: The optimal path $\mathbf{L}^*$ and the optimal cost function value $J(\mathbf{L}^*)$

Initialize $J(\boldsymbol{L}^{(0)})$ *randomly*;
Initialize precision variable with $p = 1$, $k = 0$ *e* $i = 1$;
Initialize number of points, $n = 1$;
Declare decision variables vector $\boldsymbol{L}^i$ *as non-deterministic integer variables*;
while $k \leq \eta$ **do**
    Define upper and lower limits of $\mathbf{L}$ *with directive* ASSUME, *such as* $L \in \Omega^k$;
    Describe the objective function model $J(\boldsymbol{L})$;
    do
        do
            Define the constraint $J(\boldsymbol{L}^{(i)}) < J(\boldsymbol{L}^{(i-1)})$ *with directive* ASSUME;
            Verify the satisfiability of $J_{optimal}$ *given by* $\mathrm{J}_{\mathrm{optimal}} \Leftrightarrow \mathrm{J}\left(\mathbf{L}^{(\mathrm{i})}\right) \geq \mathrm{J}\left(\mathbf{L}^{(\mathrm{i-1})}\right)$
            Update $\boldsymbol{L}^* = \boldsymbol{L}^{(i)}$ *e* $J(\boldsymbol{L}^*) = J(\boldsymbol{L}^{(i)})$ *based on the counterexample*;
            Do $i = i + 1$;
        while $\neg J_{optimal}$ *is satisfiable*;
        if $\neg J_{optimal}$ *is not consecutively satisfiable* **then**
            break
        end
        else
            Update the number of points, n;
        end
    while *TRUE*;
    Do $k = k + 1$;
    Update the set Ω^k;
    Update the precision variable, p;
end
$\boldsymbol{L}^* = \boldsymbol{L}^{(i)}$ *e* $J(\boldsymbol{L}^*) = J(\boldsymbol{L}^{(i)})$;
return $\boldsymbol{L}^*$ *e* $J(\boldsymbol{L}^*)$;

- Directive ASSUME is used for modeling the constraints set $\mathrm{J}_{\mathrm{optimal}} \Leftrightarrow \mathrm{J}\left(\mathbf{L}^{(\mathrm{i})}\right) \geq \mathrm{J}\left(\mathbf{L}^{(\mathrm{i-1})}\right)$
- Directive ASSERT is used for holding the global optimization condition
- Variable P is used to control precision and discretizes the state-space
- Optimal candidate $J\left(\boldsymbol{L}^*\right)$ is updated if a smaller cost function value is generated from the counterexample

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ACEGIO-based path planning algorithm

Algorithm 2: ACEGIO-based Path Planning Algorithm

Input : Cost function $J(\mathbf{L})$, a set of obstacles constraints $\mathbb{O}$, a set of environment constraints $\mathbb{E}$, which define Ω and a desired precision η, and a Gradient Descent function $G(\mathbf{L})$

Output: The optimal path $\mathbf{L}^*$ and the optimal cost function value $J(\mathbf{L}^*)$

Initialise $J\left(\boldsymbol{L}^{(0)}\right)$ randomly;

Initialise precision variable with $p = 1, k = 0\ i = 1$;

Initialise precision variable with $n = 1$;

Declare decision variables vector $\boldsymbol{L}^i$ as non-deterministic integer variables;

while $k \leq \eta$ **do**

  Define upper and lower limits of $\boldsymbol{L}$ with directive ASSUME, such as $L \in \Omega^k$;

  Describe the objective function model $J(\boldsymbol{L})$;

  do

    do

      Define the constraint $J\left(\boldsymbol{L}^{(i)}\right) < J\left(\boldsymbol{L}^{(i-1)}\right)$ with directive ASSUME;

      Verify the satisfiability of J_{optimal} given by $\mathrm{J}_{\text{optimal}} \Leftrightarrow \mathrm{J}\left(\mathbf{L}^{(i)}\right) \geq \mathrm{J}\left(\mathbf{L}^{(i-1)}\right)$ directive ASSERT;

      if $\neg l_{optimal}$ *is satisfiable* **then**

        Update $\boldsymbol{L}^* = \boldsymbol{L}^{(i)}$, $J(\boldsymbol{L}^*) = J\left(\boldsymbol{L}^{(i)}\right)$ based on the counterexample;

        Do $i = i + 1$;

        Update $\boldsymbol{L}^{(i)} = G\left(\boldsymbol{L}^{(i-1)}\right)$, and $J\left(\boldsymbol{L}^{(i)}\right) = J\left(\mathrm{G}\left(\boldsymbol{L}^{(i-1)}\right)\right)$;

      end

    while $\neg J_{optimal}$ *is satisfiable*;

    if $\neg J_{optimal}$ *is not consecutively satisfiable* **then**

      break ;

    else

      Update the number of points, n;

    end

  while *TRUE*;

  Do $k = k + 1$;

  Update the set Ω^k;

  Update the precision variable, p;

  Set $\boldsymbol{L}^i$ as non-deterministic integer variables;

end

$\boldsymbol{L}^* = \boldsymbol{L}^{(i-1)}$, $J(\boldsymbol{L}^*) = J\left(\boldsymbol{L}^{(i-1)}\right)$;

return $\boldsymbol{L}^* J(\boldsymbol{L}^*)$;

- The input contains Gradient descent function G(L)
- ACEGIO-GD additionally employs Gradient Descent to calculate optimal candidates iteratively
- New optimal candidates are generated either by extracting from counterexamples or by applying Gradient Descent

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Experimental Evaluation

Experimental goals:

1. **Effectiveness**: evaluate the effectiveness of the ACEGIO-based path planning algorithm
2. **Efficiency**: compare with CEGIO-based path planning algorithm
3. **State-of-the-art**: compare with other state-of-the-art approaches

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Experimental Setup

Tools for executing CEGIO-based path planning algorithm and ACEGIO-based path planning algorithm:

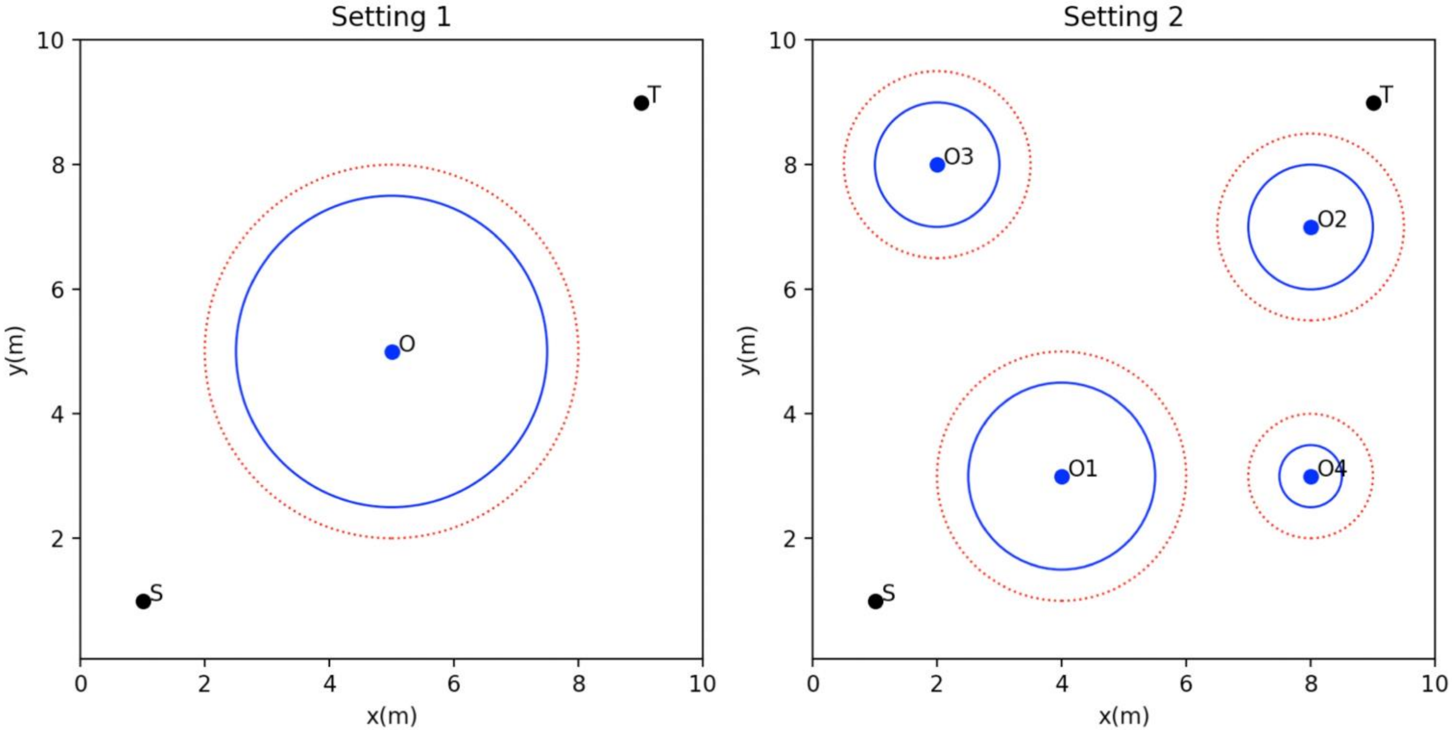
- Model checker: ESBMC 6.4.0 64-bits
- SMT solver: Boolector 3.0
- 2.3 GHz OCTA Intel Core i9 processor with 16GB of RAM, running macOS Catalina 10.15.6 64-bits

Tools for executing GA-based path planning algorithm and PSO-based path planning algorithm:

- Matlab R2021a

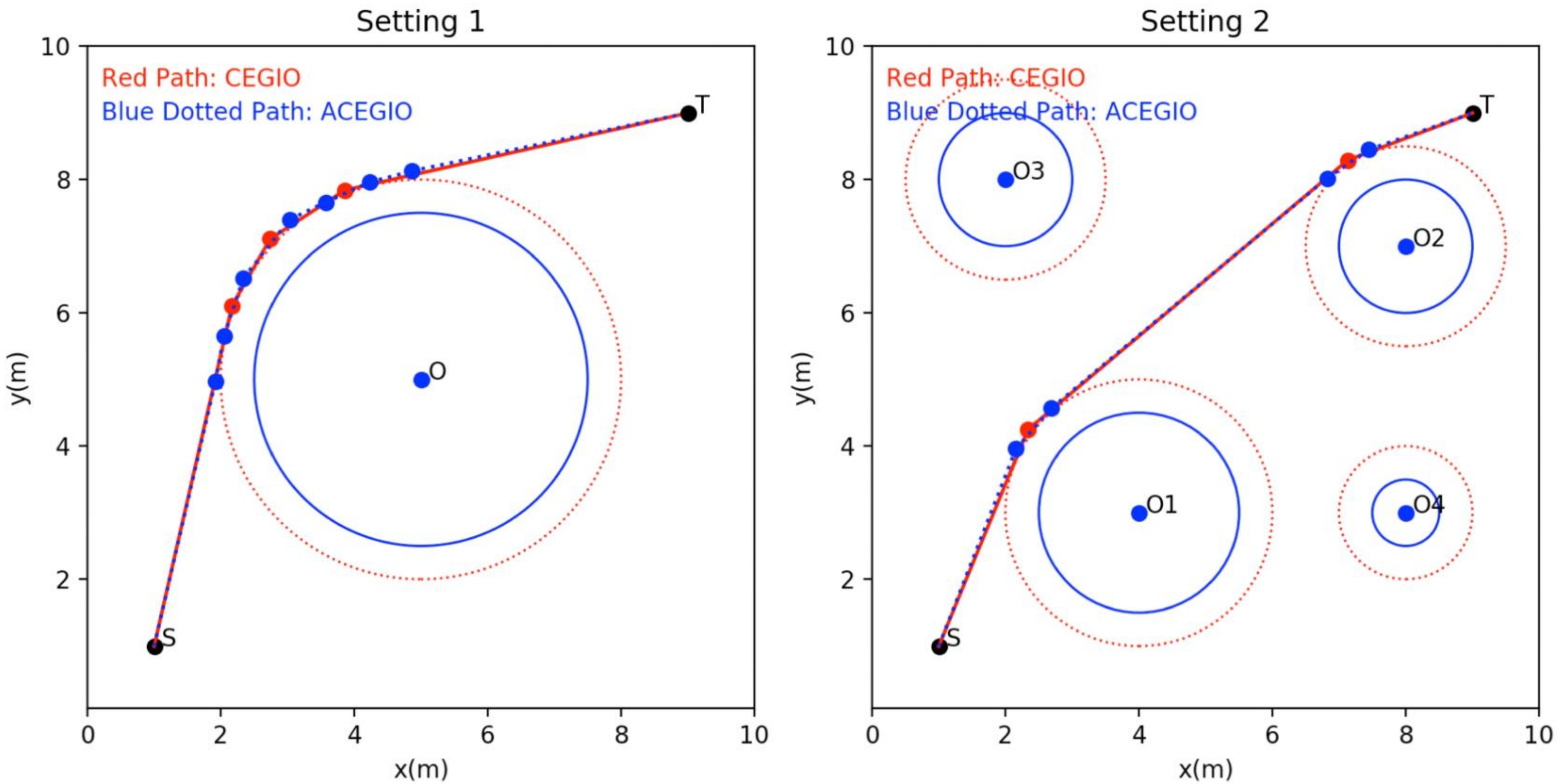
Experimental environment settings

- Motion space is modeled as square
- Obstacles are modeled as circles (blue circles), safety margin is represented by red circles
- Point S (1,1) is the starting point and Point T (9,9) is the target point



Experimental Results

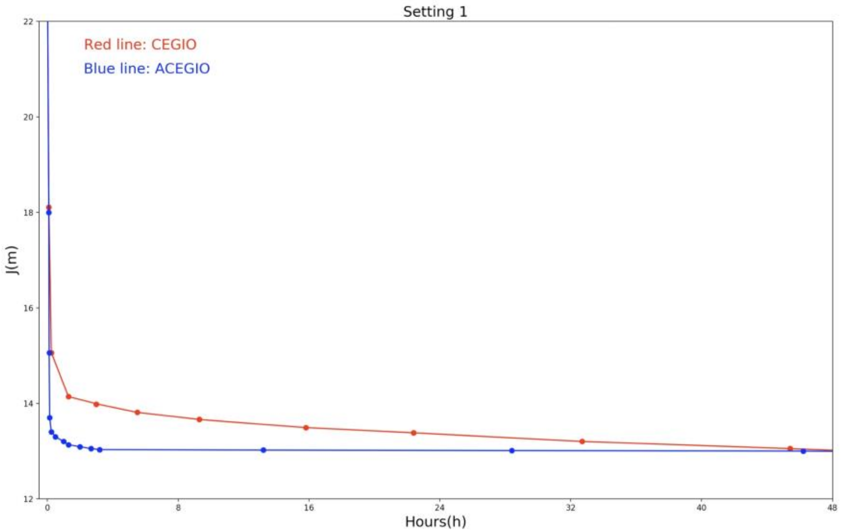
Evaluate CEGIO-based path planning algorithm and ACEGIO-based path planning algorithm

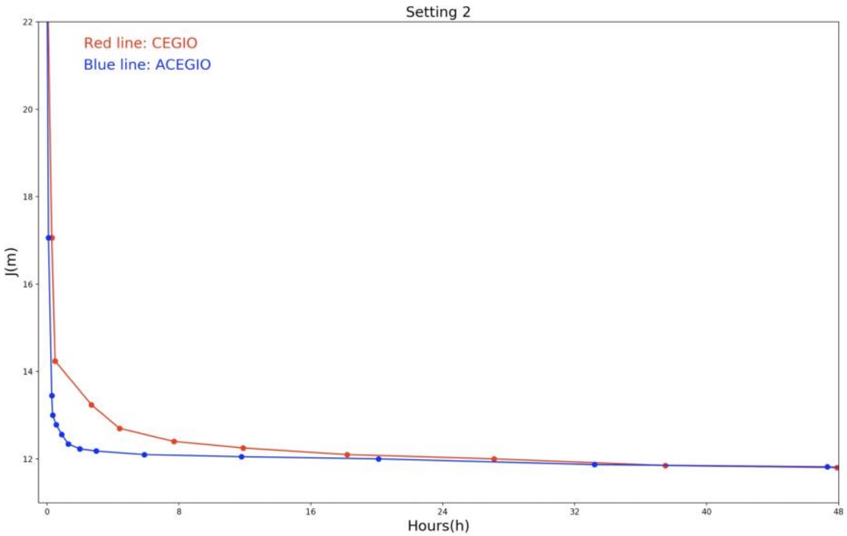


EG1: ACEGIO-based path planning algorithm can generate the global optimal path or a path close to the global optimal path

Experimental Results

Compare CEGIO-based path planning algorithm and ACEGIO-based path planning algorithm





- Reduction trend of cost function values which were obtained by the evaluated algorithms
- Horizontal axis represents the time of optimizing the path planning problem
- Cost function values are shown in the vertical axis

EG2: ACEGIO-based path planning algorithm generate paths closer to the global optimality with less execution time comparing to CEGIO-based path planning algorithm

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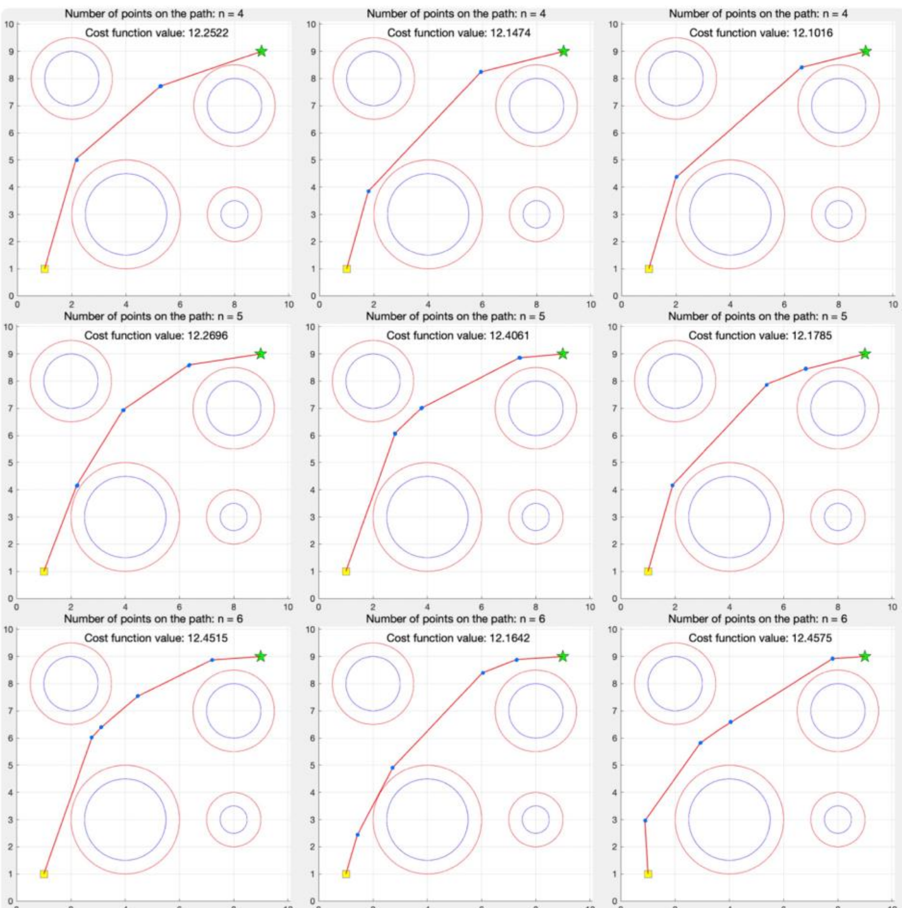
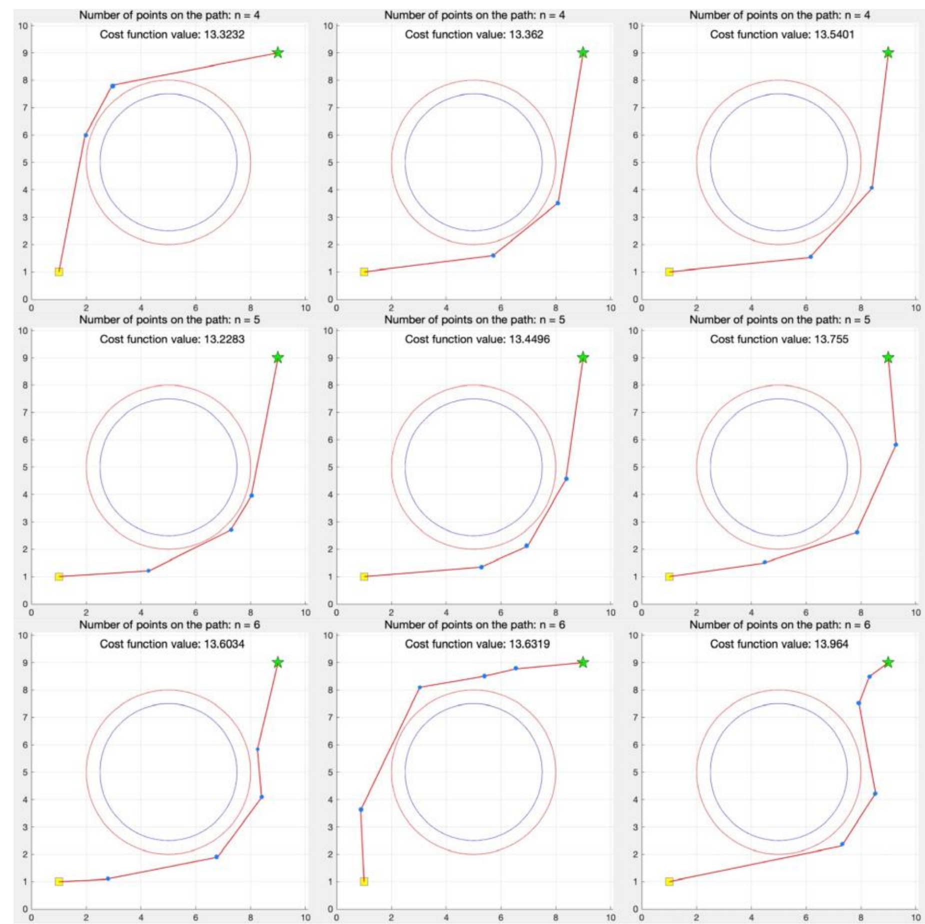
Experimental Results

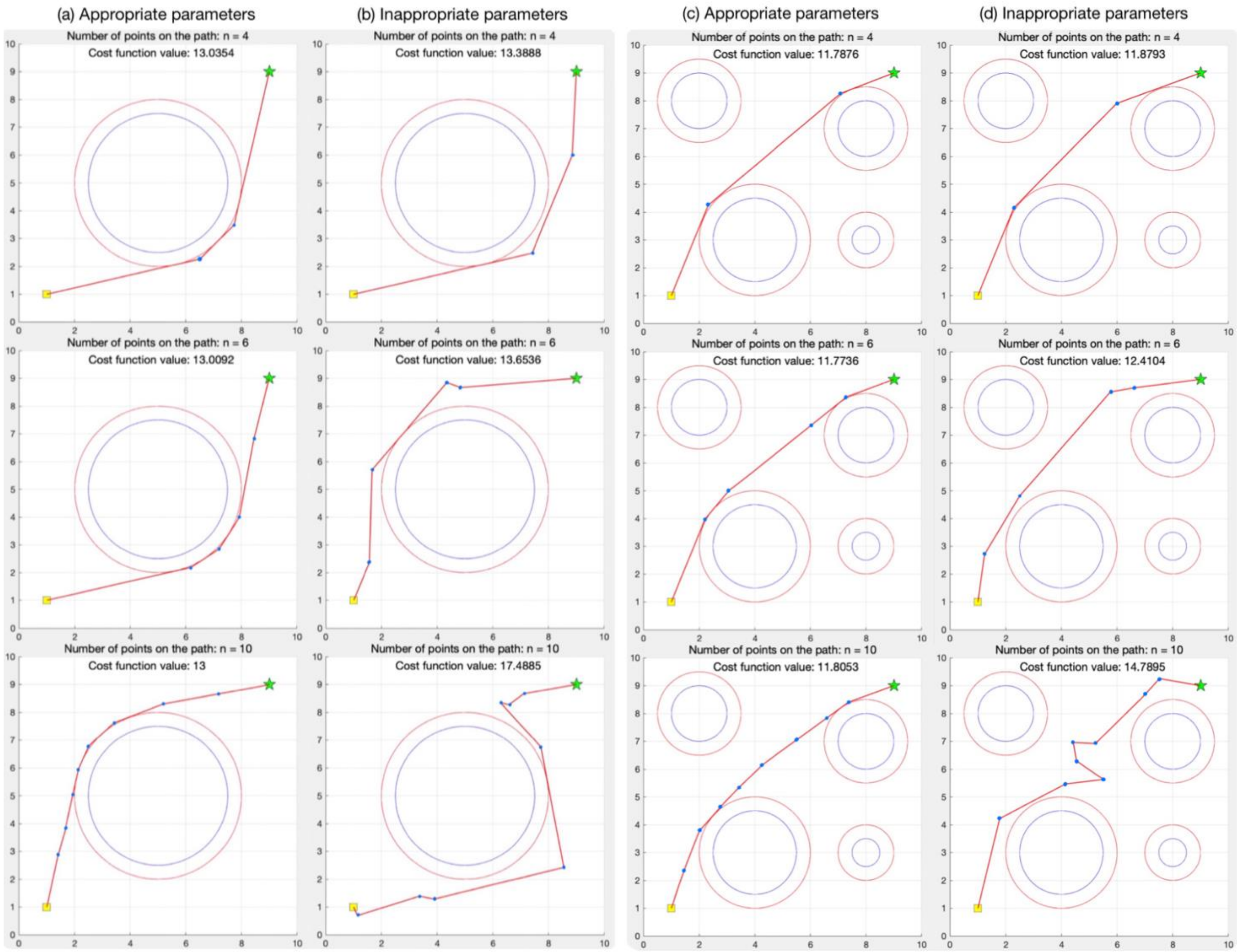
Evaluate GA-based path planning algorithm





Experimental Results

Evaluate PSO-based path planning algorithm



Experimental Results

Compare among path planning algorithms based on ACEGIO, GA and PSO

1. **GA-based path planning algorithm**: much faster, while **ACEGIO-based path planning algorithm**: more effective and more stable
2. **PSO-based path planning algorithm**: stable, fast and reliable, but not easy to tune parameters. **ACEGIO-based path planning algorithm:** much easier to tune.

EG3: If compared to path planning algorithms based on GA and PSO, the execution time of the proposed algorithm is relatively high, whereas its performance is stable, reliable and robust

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Conclusions

- We presented a novel mobile robot path planning algorithm, which relies on the ACEGIO-GD algorithm to solve the optimal path planning problem
- The proposed algorithm can generate optimal paths with significantly shorter execution time than the original CEGIO-based path planning algorithm
- If compared to GA-based path planning algorithm, ACEGIO-based path planing is slow but more stable and reliable
- If compared to PSO-based path planning algorithm, ACEGIO-based path planning is slow but more robust

Future work

- develop other auxiliary algorithms to assist CEGIO
- explore the best auxiliary algorithm for CEGIO to solve optimal path planning problems