FEDERAL UNIVERSITY OF RORAIMA
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Model Checking Embedded C Software using *k*-Induction and Invariants

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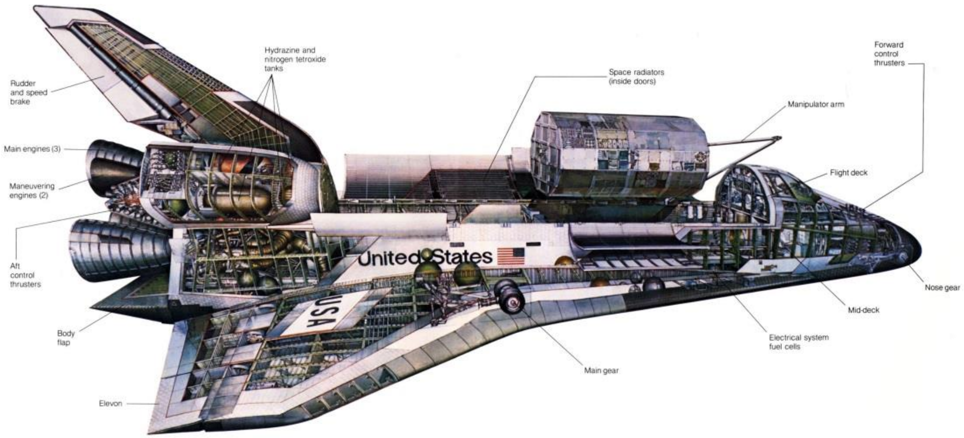
Agenda

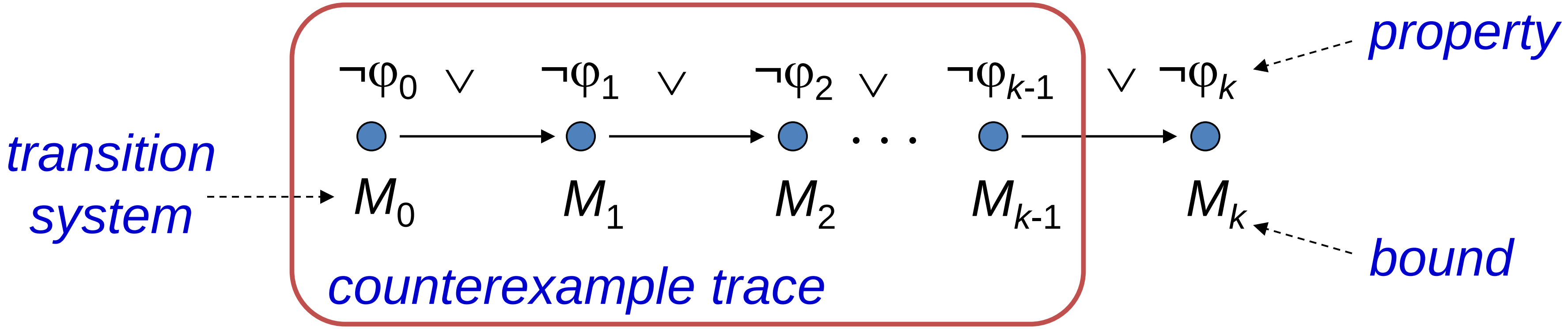
1. Introduction
2. Background
3. Proposed Method
4. Experimental Evaluation
5. Related Work
6. Conclusions and Future Work

Software Applications



Bounded Model Checking (BMC)

Basic Idea: check negation of given property up to given depth



property

$\neg\varphi_0 \lor \neg\varphi_1 \lor \neg\varphi_2 \lor \neg\varphi_{k-1} \lor \neg\varphi_k$

transition system

$M_0 \quad M_1 \quad M_2 \quad \ldots \quad M_{k-1} \quad M_k$

bound

counterexample trace

- transition system *M* unrolled *k* times
  - for programs: loops, arrays, …
- translated into verification condition ψ such that

ψ satisfiable iff φ has counterexample of max. depth *k*

- has been applied successfully to verify (embedded) software

Bounded Model Checking (BMC)

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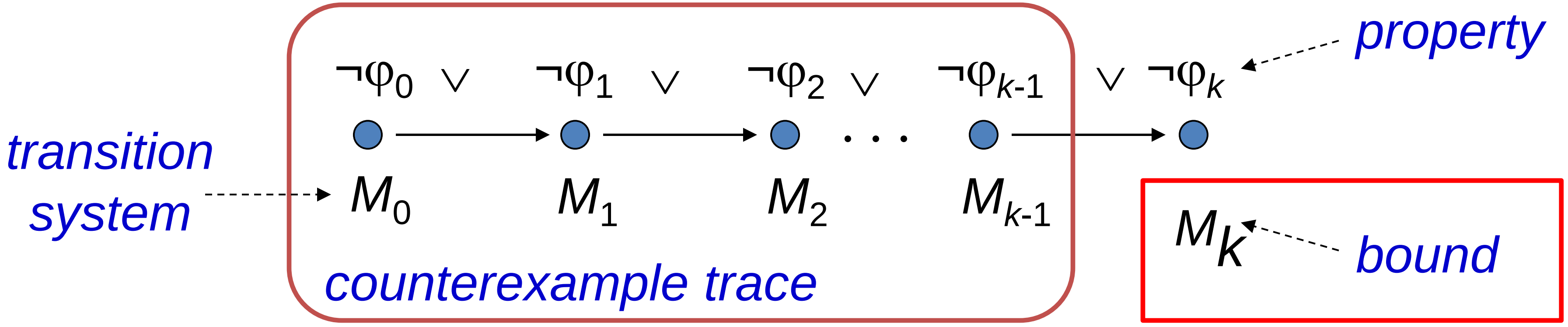
$\neg\varphi_0 \vee \neg\varphi_1 \vee \neg\varphi_2 \vee \cdots \neg\varphi_{k-1} \vee \neg\varphi_k$

$M_0 \rightarrow M_1 \rightarrow M_2 \cdots M_{k-1} \rightarrow M_k$

- BMC techniques limit the visited regions of data structures (*e.g.*, arrays) and the number of loop iterations.
- BMC tools are susceptible to exhaustion of time or memory limits for programs with **loops**.

Example

$$S_n = \sum_{i=1}^{n} a = na, n \geq 1$$

```
int main(int argc, char **argv)
{
 long long int i = 1, sn
 unsigned int n;
 assume(n>=1);
 while (i<=n) {
    sn = sn+a;
    i++;
 }
 assert(sn==n*a);
}

```

Bound ***loop***

Example

$$S_n = \sum_{i=1}^{n} a = na, n \geq 1$$

```
int main(int argc, char **argv)
{
 long long int i = 1, sn
 unsigned int n;
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    i++;
 }
 assert(sn==n*a);
}

```

Bound ***loop***

For a 32 bits integer, the loop will be unfolded $2^n - 1\ times =$ **4,294,967,295 times**

Difficulties in proving the correctness of programs with loops in BMC

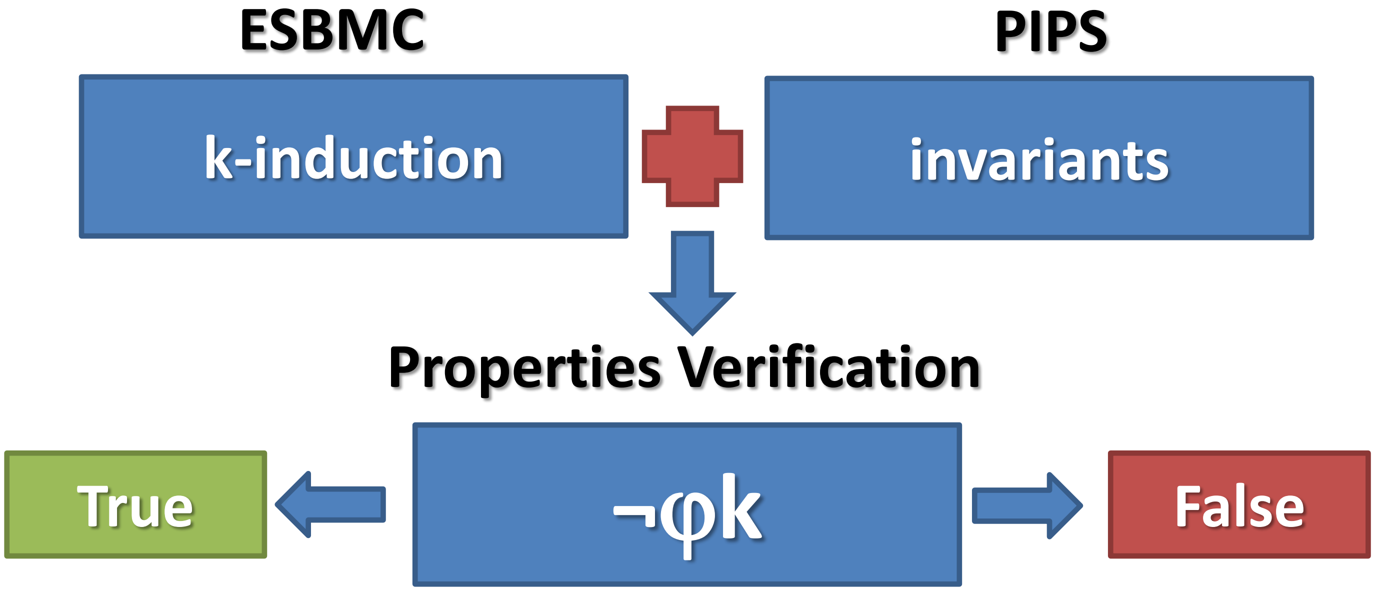
- BMC techniques can falsify properties up to a given depth *k*
  - they can prove correctness only if an upper bound of *k* is known (**unwinding assertion**)

Solution: handles such (unbounded) problems using proof by induction

- k-induction has been successfully combined with continuously-refined invariants
  - to prove that (restricted) C programs do not contain data races (Donaldson et al., 2010)
  - in hardware verification (Eén and Sörensson, 2003)

Difficulties in proving the correctness of programs with loops in BMC

- This paper contributes:
    - a new algorithm to prove correctness of C programs
    - combining k-induction with invariants
    - in a completely automatic way



ESBMC

PIPS

k-induction

invariants

Properties Verification

True

¬φk

False

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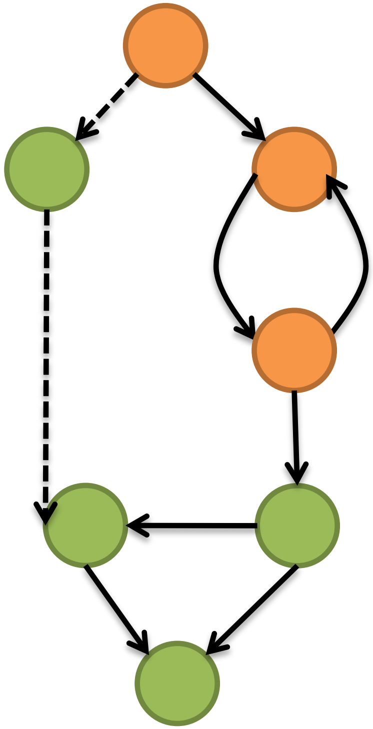
Efficient SMT-Based Bounded Model Checking - ESBMC

ESBMC is a bounded model checker for embedded ANSI-C software based on SMT (Satisfiability Modulo Theories) solvers, which allows:

- ✓ Out-of-bounds array indexing;
- ✓ Division by zero;
- ✓ Pointers safety
- ✓ Dynamic memory allocation;
- ✓ Multi-Threaded software
- ✓ Data races;
- ✓ Deadlocks;
- ✓ Underflow e Overflow;

Program Invariants

Invariants are properties of program variables and relationships between these variables in a specific line of code which called program point.



`n = size(b)`

```
i := 0;
s := 0;
while i ≠ n{
   s := s+b[i];
   i := i+1;
}
```

`s = sum(b[0..i-1])`

`s = sum(b)`

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Induction-based Verification of C programs using Invariants

k-induction checks for each step k

- **base case (*$base_k$*):** find a counter-example with up to *k* loop unwindings (plain BMC)
- **forward condition (*fwd_k*):** check that *P* holds in all states reachable within *k* unwindings
- **inductive step (*$step_k$*):** check that whenever *P* holds for *k* unwindings, it also holds after next unwinding
  - havoc state
  - run *k* iterations
  - assume invariant
  - run final iteration

Induction-based Verification of C programs using Invariants

```
k = 1; force_basecase = FALSE; last_result = UNKNOWN;
while k <= max_iterations do
if force_basecase then
    k = k + 5;
    if base_case(P, ϕ, k) then
        show counterexample s[0..k];
        return FALSE;
    else
      if force_basecase then return last_result
      k=k+1
      if forward_condition(P, ϕ, k) then
         force_basecase = TRUE;  last_result = TRUE;
      else
        if inductive_step(P, ϕ, k) then
          force_basecase = TRUE;  last_result = TRUE;

return UNKNOWN
```

Induction-based Verification of C programs using Invariants

input: program P and safety property ϕ

output: true, false, or unknown

```
    k = k + 5;
    if base_case(P, $\phi$, k) then
        show counterexample s[0..k];
        return FALSE;
    else
      if force_basecase then return last_result
      k=k+1
      if forward_condition(P, $\phi$, k) then
         force_basecase = TRUE;  last_result = TRUE;
      else
        if inductive_step(P, $\phi$, k) then
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return UNKNOWN
```

Induction-based Verification of C programs using Invariants

```
k = 1; force_basecase = FALSE; last_result = UNKNOWN;
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    if base_case(P, $\phi$, k) then
        show counterexample s[0..k];
        return FALSE;
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      k=k+1
      if forward_condition(P, $\phi$, k) then
        force_basecase = TRUE;  last_result = TRUE;
      else
        if inductive_step(P, $\phi$, k) then
         force_basecase = TRUE;  last_result = TRUE;

return UNKNOWN
```
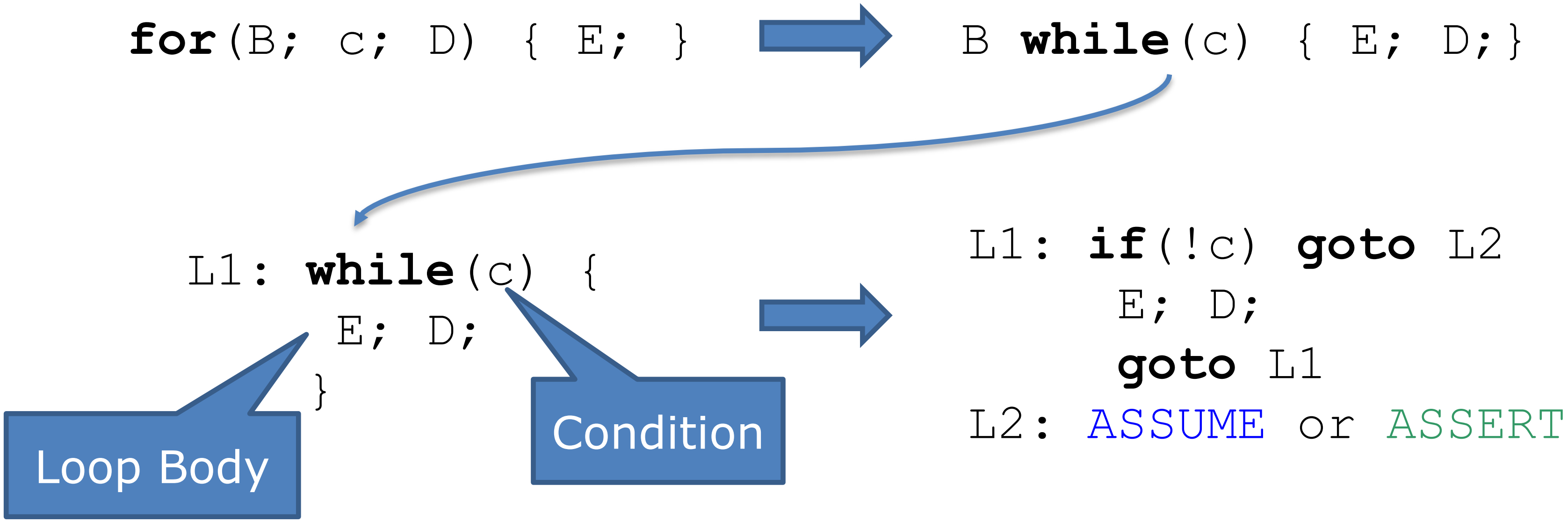

Rechecking of the result.
It is needed due to the
inclusion of invariants.

Avoid incorrect
exploration

Loop-free Programs ($base_k$ and fwd_k)

A loop-free program is represented by a **straight-line program** (without loops) using *if*-statements



```
for(B; c; D) { E; }
```

```
B while(c) { E; D;}
```

```
L1: while(c) {
      E; D;
    }
```

Loop Body

Condition

```
L1: if(!c) goto L2
      E; D;
      goto L1
L2: ASSUME or ASSERT
```

Loop-free Programs (*step$_k$*)

In the inductive step, loops are converted into:

the code to remove redundant states

`while(c) { E; }` → `A while(c) { S; E; U; } R;`

- **A:** assigns **non-deterministic values** to all loops variables (the state is havocked before the loop)
- **c:** is the **halt condition** of the loop
- **S: stores the current state** of the program variables before executing the statements of E
- **E:** is the actual **code inside the loop**
- **U: updates all program variables** with local values after executing E

Program Transformation

```
k = 1; force_basecase = FALSE; last_result = UNKNOWN;
while k <= max_iterations do
if force_basecase then
    k = k + 5;
    if base_case(P, $\phi$, k) then
        show countere      e s[0..k];



                                   ret

      if forward_condition(P, $\phi$, k) then
         force_basecase = TRUE;  last_result = TRUE;
      else
        if inductive_step(P, $\phi$, k) then
          force_basecase = TRUE;  last_result = TRUE;

return UNKNOWN
```

$I \wedge T \wedge \sigma \Rightarrow \phi$

I : initial condition
T : transition relation of P′
σ : termination condition
ϕ : safey property

inserts **unwinding assumption** after each loop

Program Transformation

I : initial condition
T : transition relation of P′
σ : termination condition
ϕ : safey property

$I \wedge T \wedge \sigma \Rightarrow \phi$

```
                          FALSE; last_result = UNKNOWN;
                          s do


    if base_case(P, $\phi$, k) then
        show counterexample s[0..k];
        return FALSE;
    else
      if force_basecase then retu
      k=k+1
      if forward_condition(P, $\phi$,
         force_basecase = TRUE;  last_result = TRUE;
      else
        if inductive_step(P, $\phi$, k) then
          force_basecase = TRUE;  last_result = TRUE;

return UNKNOWN
```

$I \wedge T \Rightarrow \sigma \wedge \phi$

inserts **unwinding assertion** after each loop

Program Transformation

I : initial condition
T : transition relation of P’
σ : termination condition
ϕ : safey property

```
FALSE; last_result = UNKNOWN;
s do


    if base_case(P, $\phi$, k) then
        show counterexample s[0..k];
        return FALSE;
    else
      if force_basecase then return last_result
      k=k+1
      if forward_condition(P, $\phi$, k) then
         force_basecase = TRUE;  last_result = TRUE;
      else
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return UNKNOWN
```

$I \wedge T \wedge \sigma \Rightarrow \phi$

$I \wedge T \Rightarrow \sigma \wedge \phi$

γ: transition relation of P’

$\boxed{\gamma \wedge \sigma \Rightarrow \phi}$

Program Transformation

```
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while k <= max_iterations do
if force_basecase then
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    if base_case(P, $\phi$, k) then
        show counterexample s[0..k];
        return FALSE;
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      if force_basecase then return last_result
      k=k+1
      if forward_condition(P, $\phi$, k) then
         force_basecase = TRUE;  last_result = TRUE;
      else
        if inductive_step(P, $\phi$, k) then
          force_basecase =                          = TRUE;

return UNKNOWN
```

$I \wedge T \wedge \sigma \Rightarrow \phi$

$I \wedge T \Rightarrow \sigma \wedge \phi$

$\boxed{\gamma \wedge \sigma \Rightarrow \phi}$

unable to **falsify** or **prove** the property

Invariant Generation

To infer program invariants, we adopted the PIPS tool

- It is an interprocedural source-to-source compiler framework for C and Fortran programs
- It relies on a polyhedral abstraction of program behavior

```
...
// P(i,k,n0,n1,n2){i==0, 0<=k, n0<=k, n0+n1<=k, n0+n1+n2<=k,
                   n0+n2<=k, n1<=k, n1+n2<=k, n2<=k}

while (i<n2) {
   //  P(i,k,n0,n1,n2) {0<=i, n0+n1+n2<=i+k, n0+n2<=i+k,
   //                   n1+n2<=i+k, n2<=i+k, i+1<=n2}

   i++;
...
```

polyhedral invariants are propagated along with instructions

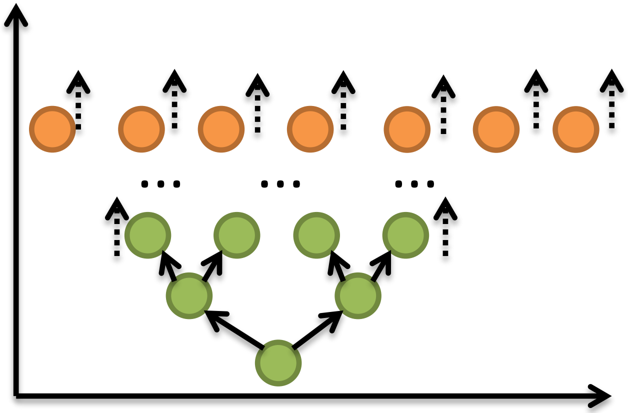
Invariant Generation

PIPS Invariant Translation

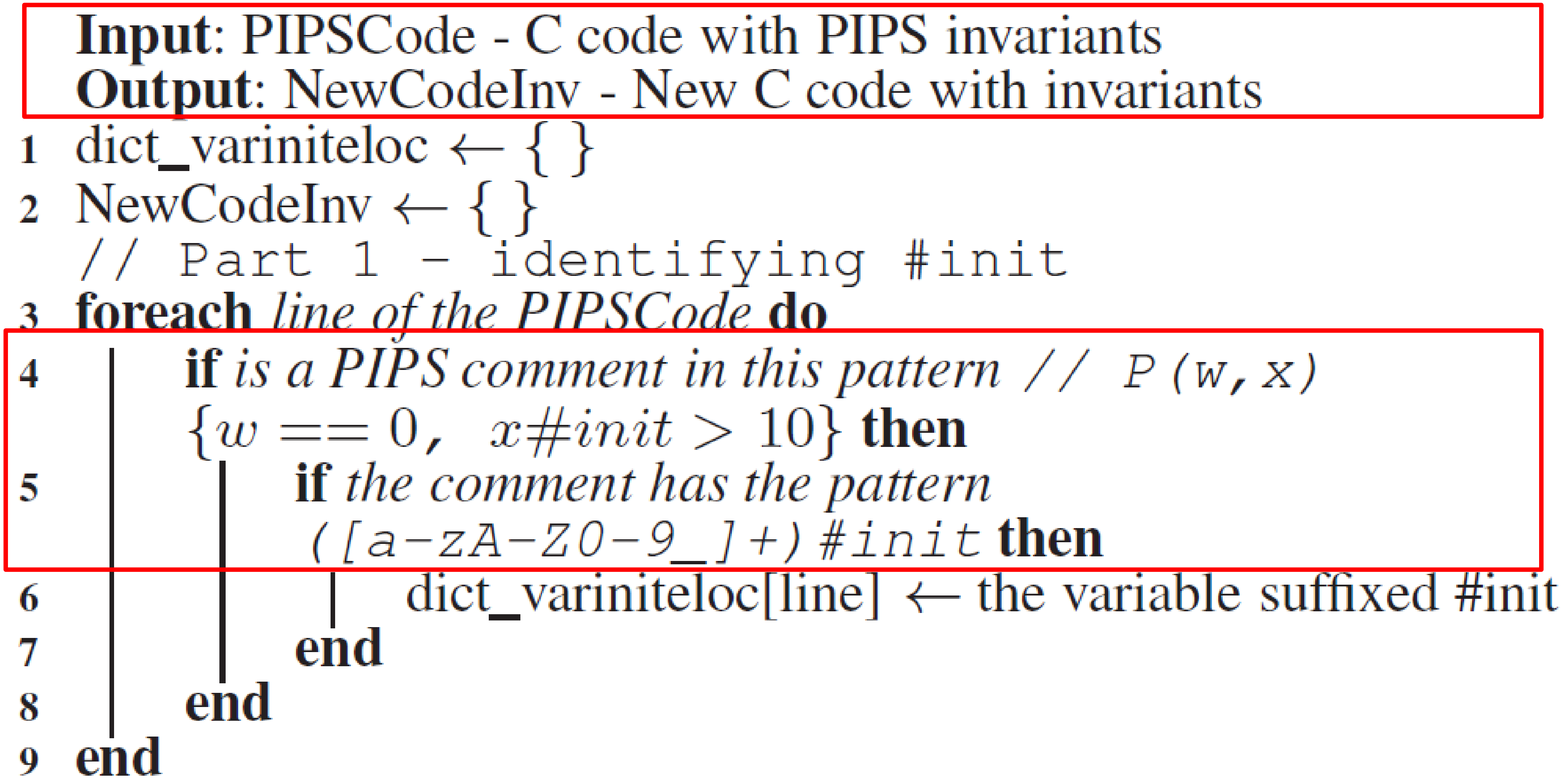
- Mathematical expressions,e.g, $2j < 5t$
- Invariants with $\#init$ suffix that is used to distinguish the old value to the new value

The invariants are translated and instrumented into the program as assume statements

- `assume(expr)` to limit possible values of the variables that are related to the invariants

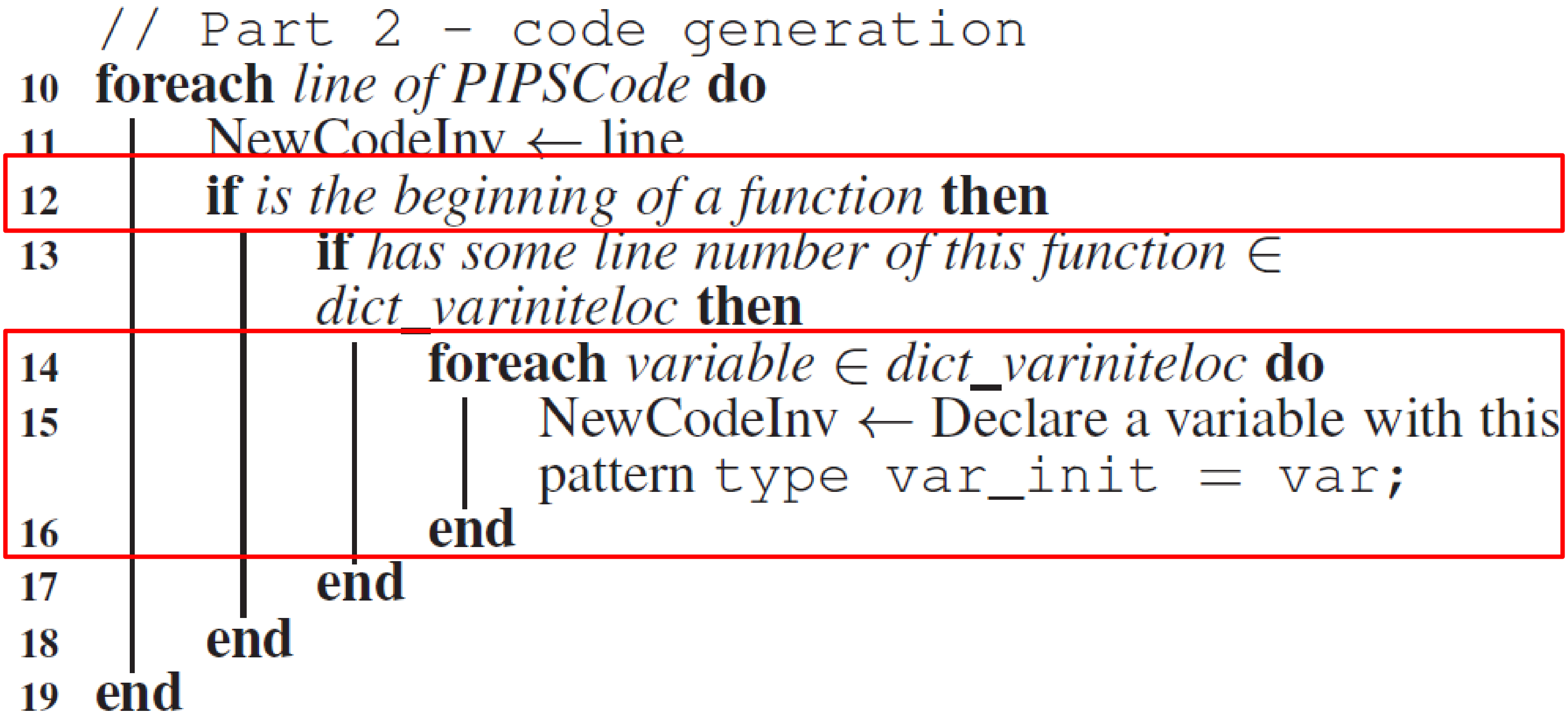


Translation algorithm of invariants



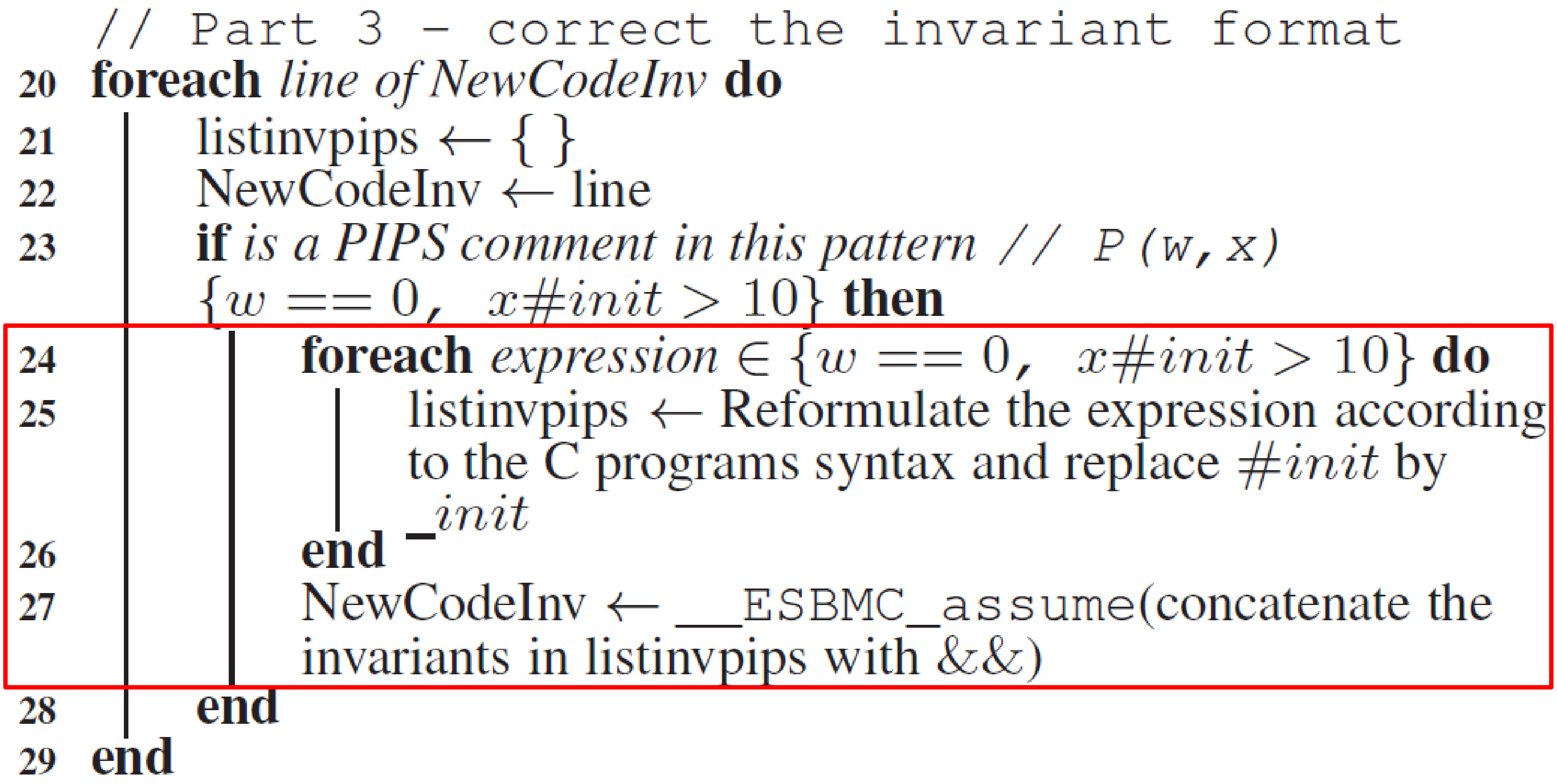
```
Input: PIPSCode - C code with PIPS invariants
Output: NewCodeInv - New C code with invariants
dict_variniteloc ← { }
NewCodeInv ← { }
// Part 1 - identifying #init
foreach line of the PIPSCode do
    if is a PIPS comment in this pattern // P(w,x) $\{w == 0,\ x\#init > 10\}$ then
        if the comment has the pattern ([a-zA-Z0-9_]+)#init then
            dict_variniteloc[line] ← the variable suffixed #init
        end
    end
end
```

Translation algorithm of invariants



```
// Part 2 - code generation
foreach line of PIPSCode do
    NewCodeInv ← line
    if is the beginning of a function then
        if has some line number of this function ∈ dict_variniteloc then
            foreach variable ∈ dict_variniteloc do
                NewCodeInv ← Declare a variable with this pattern type var_init = var;
            end
        end
    end
end
```

Translation algorithm of invariants



```
// Part 3 - correct the invariant format
foreach line of NewCodeInv do
    listinvpips ← { }
    NewCodeInv ← line
    if is a PIPS comment in this pattern // P(w,x) {$w == 0,\ x\#init > 10$} then
        foreach expression ∈ {$w == 0,\ x\#init > 10$} do
            listinvpips ← Reformulate the expression according to the C programs syntax and replace $\#init$ by $\_init$
        end
        NewCodeInv ← __ESBMC_assume(concatenate the invariants in listinvpips with &&)
    end
end
```

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Planning and Designing the Experiments

Goal: Analyzing the ability of DepthK to **verify** a wide variety of safety properties in C programs.

✓ The experiments are conducted on an Intel Xeon CPU E5 – 2670 CPU, 2.60GHz, 115GB RAM with Linux OS

✓ The time limit to the verification is 15 min

✓ Memory consumption limit to 15 GB

DepthK tool is available at https://github.com/hbgit/depthk

Planning and Designing the Experiments

- ✓ **142 ANSI-C** programs of the **SV-COMP 2015** (Beyer, 2015);
- ✓ **34 ANSI-C** programs used in **embedded systems:**
  - Powerstone (Scott et al., 1998)
  - SNU real-time (SNU, 2012)
  - WCET (MRTC, 2012)
- ✓ Comparison with the tools:
  - CPAChecker SVN v15596 (Beyer e Keremoglu, 2011)
  - CBMC v5.0 with *k*-induction (Clarke et al., 2004a)
  - ESBMC v1.25.2 with *k*-induction (Cordeiro et al., 2012)

Planning and Designing the Experiments

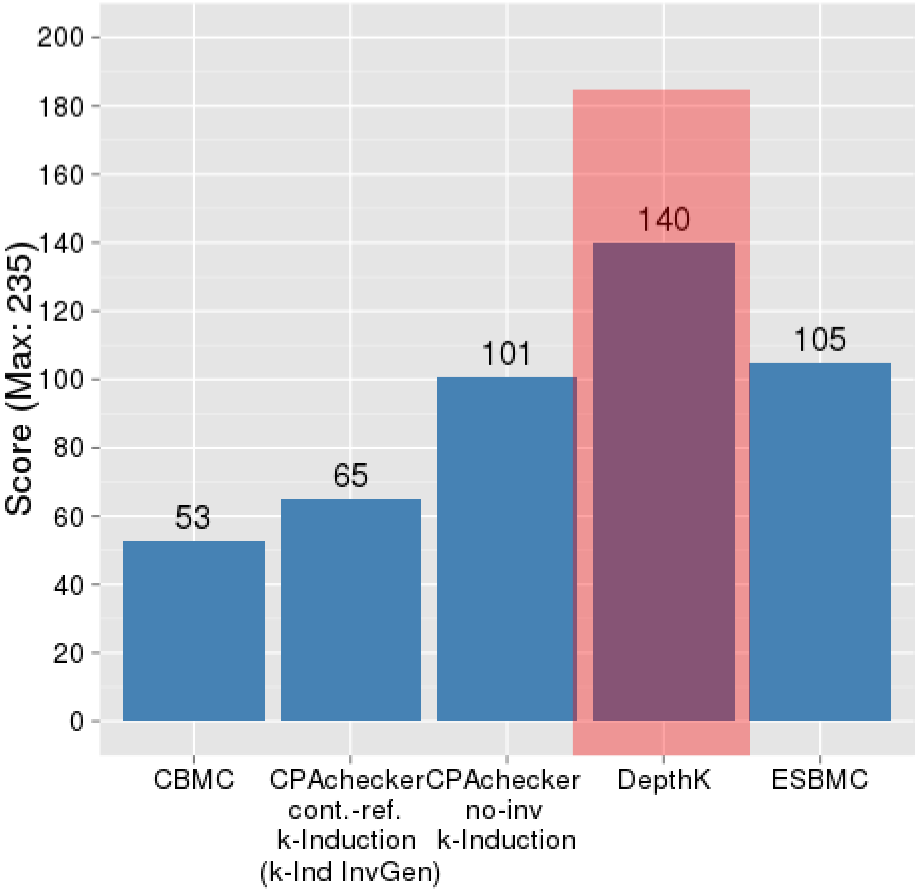
In the experiments, we collect the data:

- ✓ Correct Results
- ✓ False Incorrect
- ✓ True Incorrect
- ✓ Unknown
- ✓ Time

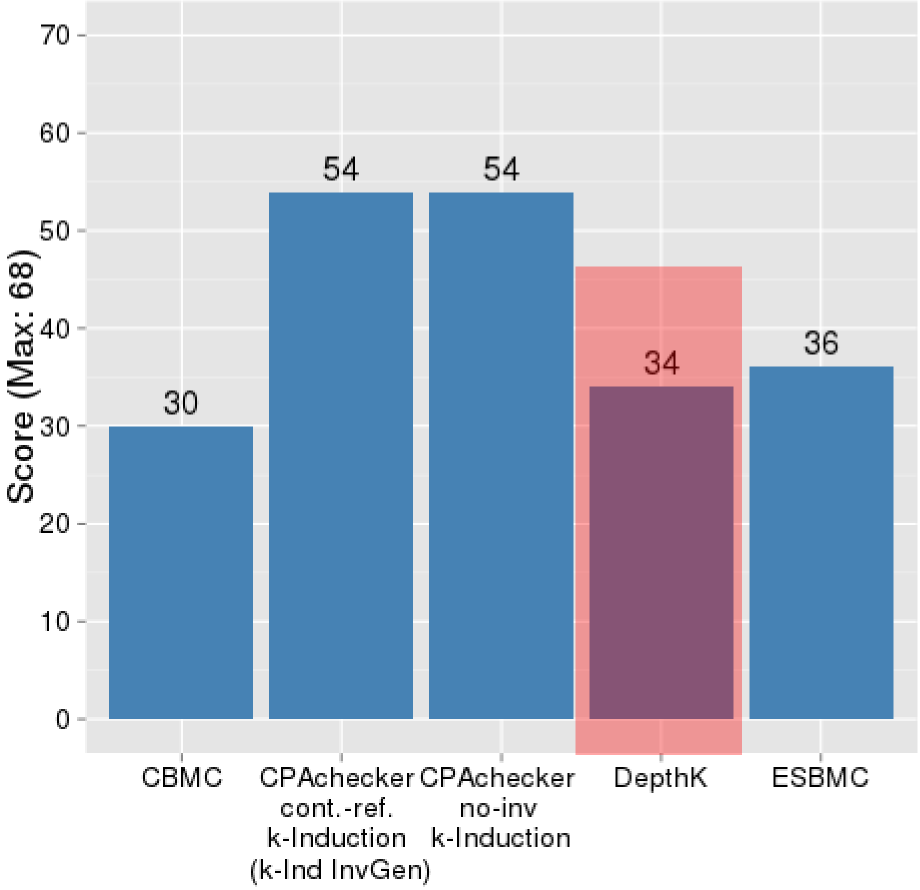
To evaluate we adopted the same scoring scheme that is used in SVCOMP 2015, e.g., **12 scores are subtracted** for every wrong safety proof (True Incorrect).

Experimental Results

SV-COMP: Loops benchmarks

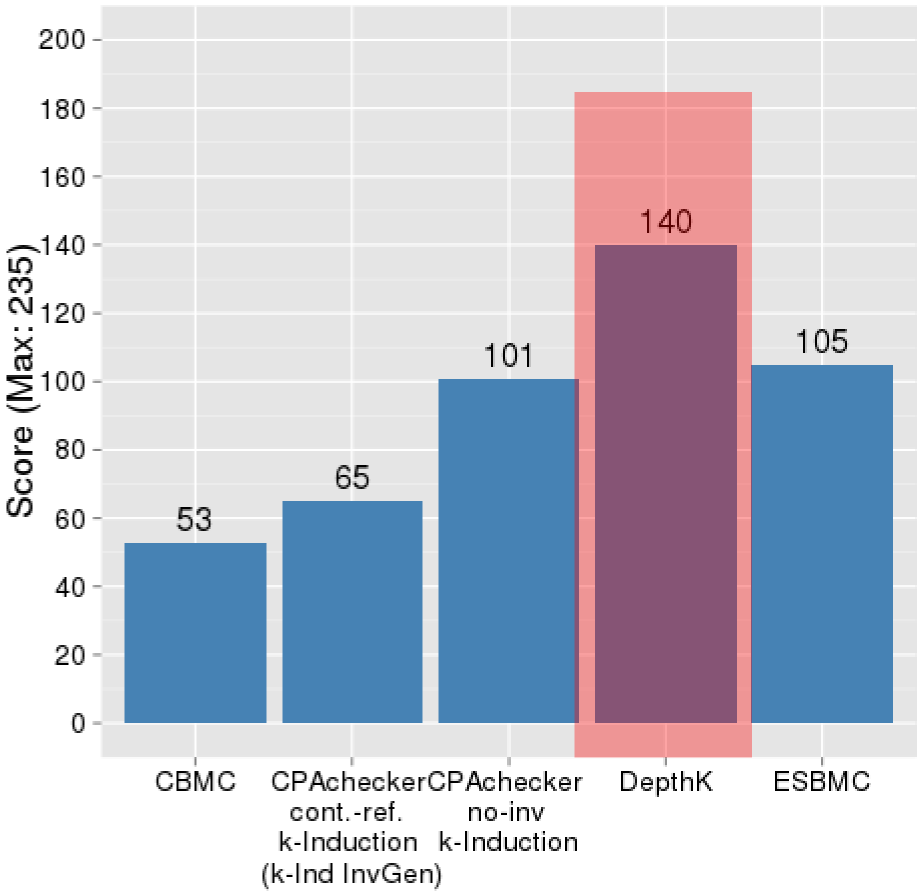


Embedded systems programs

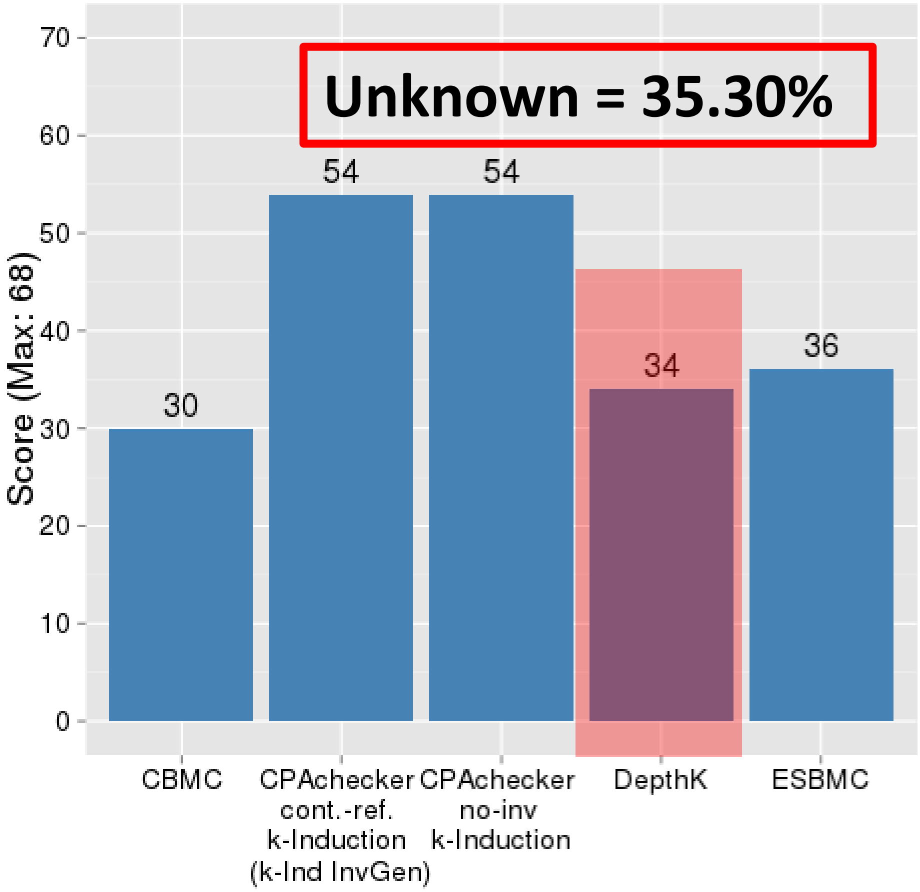


Experimental Results

SV-COMP: Loops benchmarks



Embedded systems programs

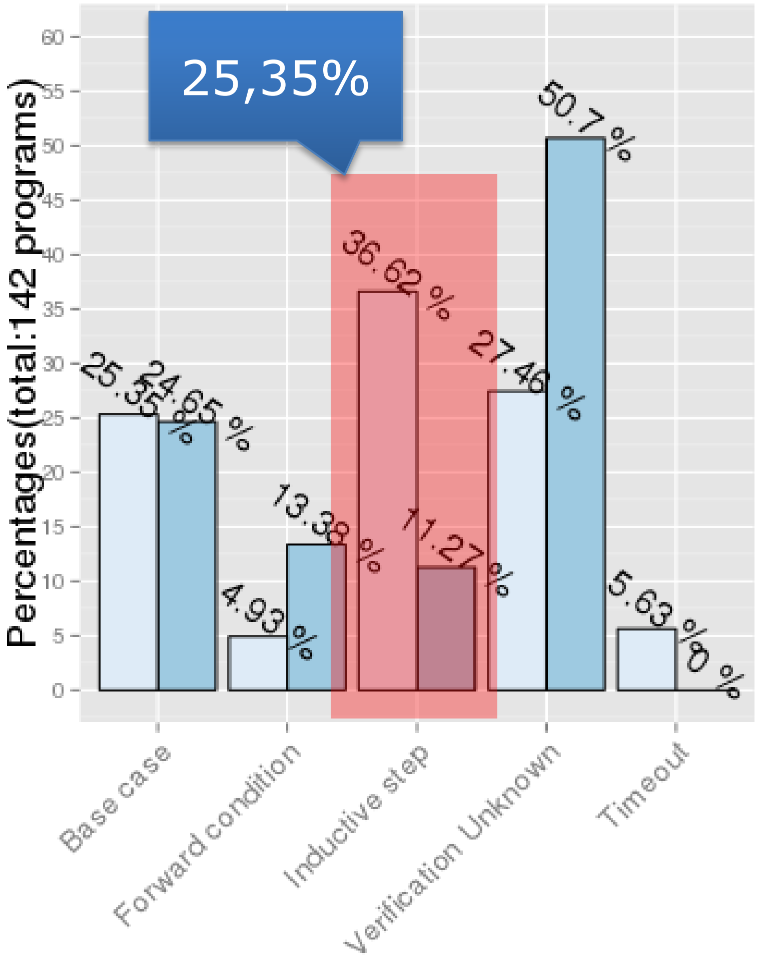


Unknown = 35.30%

Experimental Results

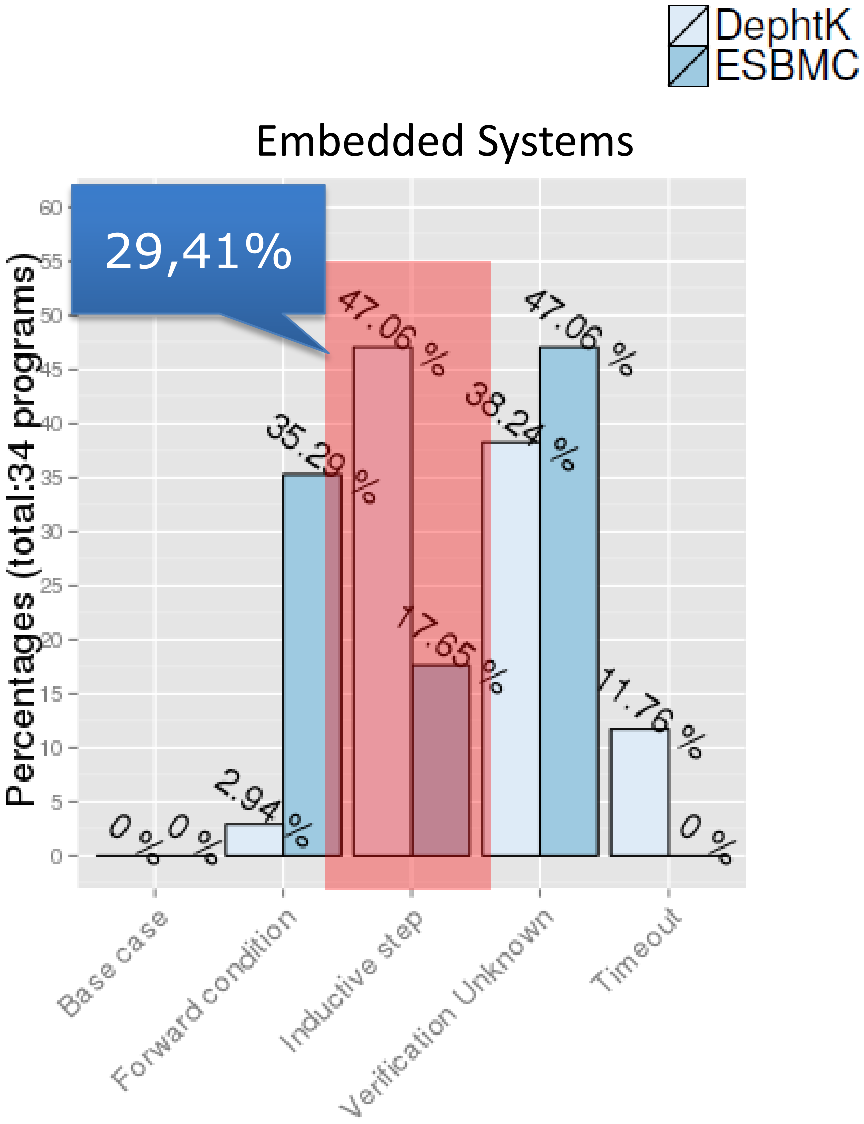
- ✓ *K*-induction steps

SV-COMP



25,35%

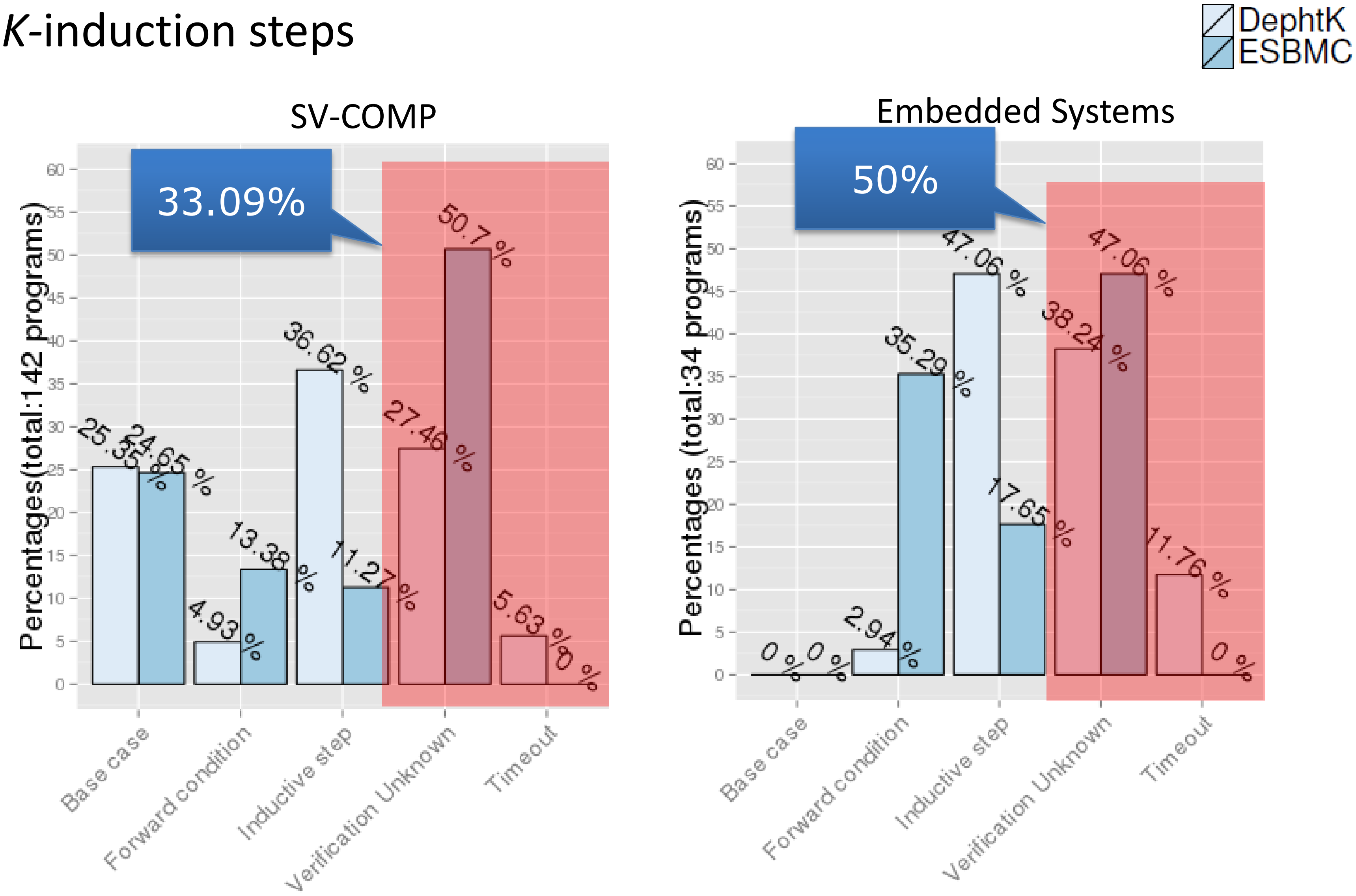
Embedded Systems



29,41%

Experimental Results

- ✓ *K*-induction steps



SV-COMP

33.09%

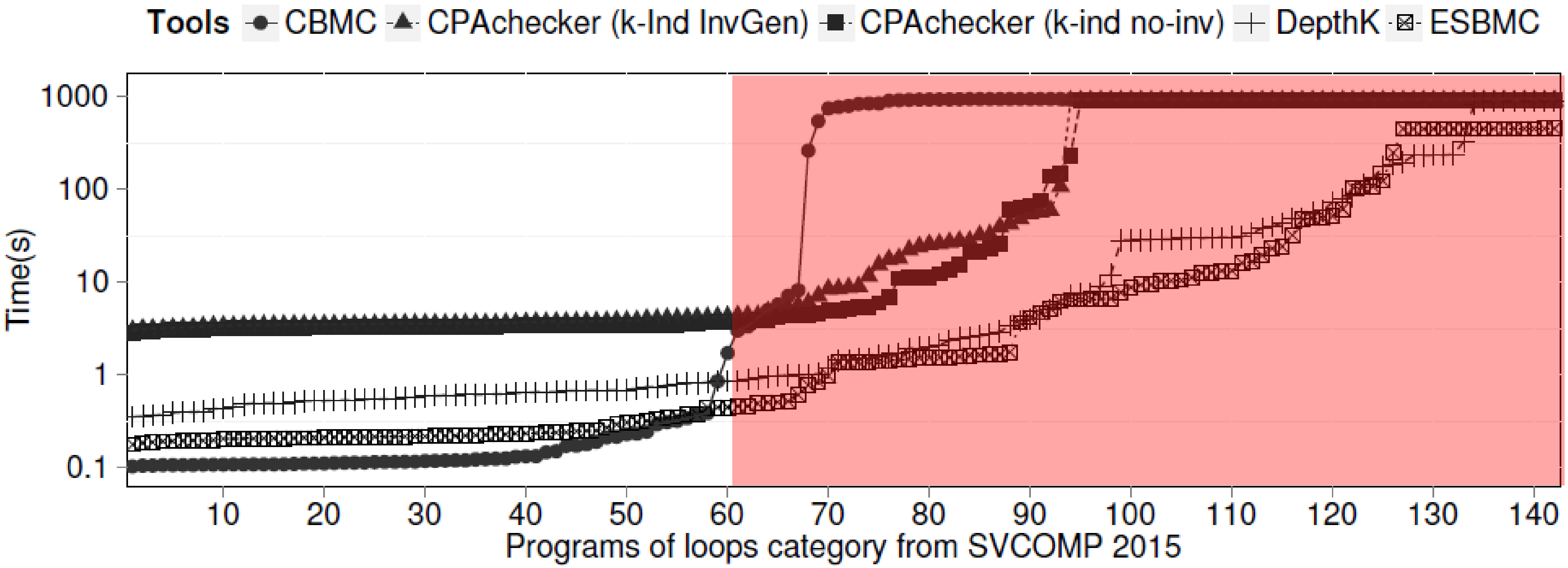
Embedded Systems

50%

Experimental Results

The verification time:

✓ DepthK is usually **faster** than the other tools, **except for ESBMC**

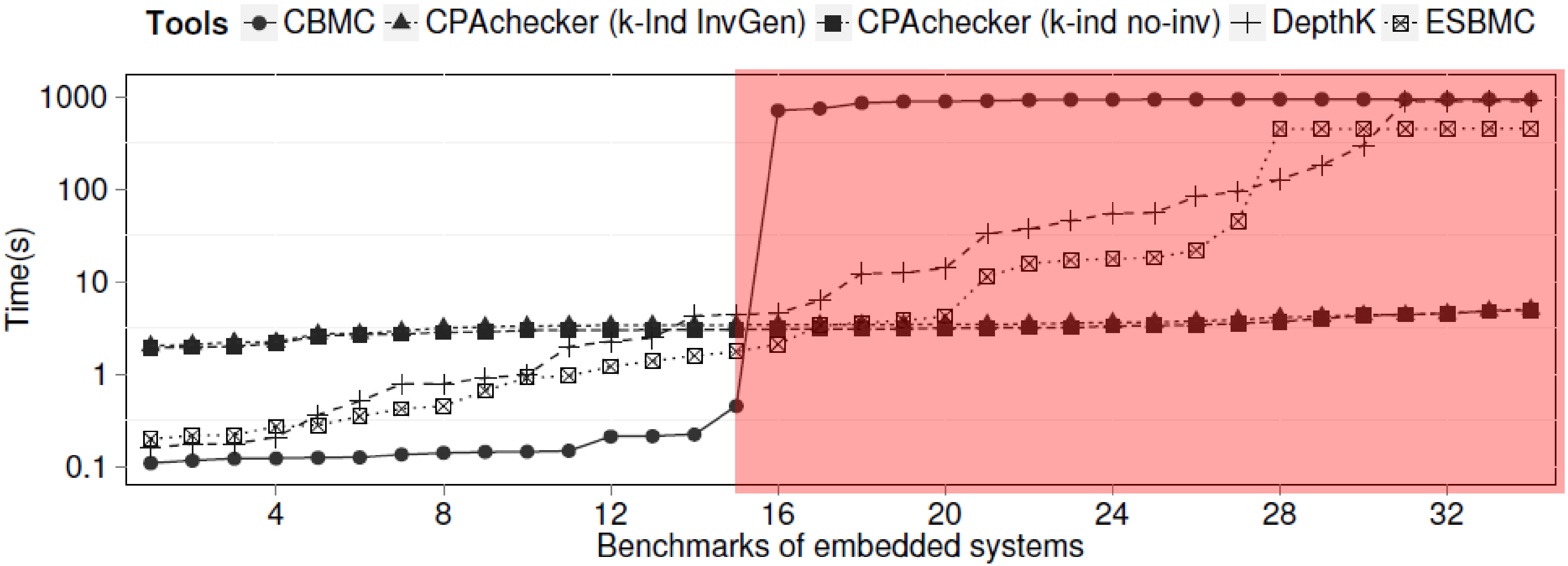


Experimental Results

The verification time:

✓ DepthK is **only faster** than CBMC

35.30% de Unknown

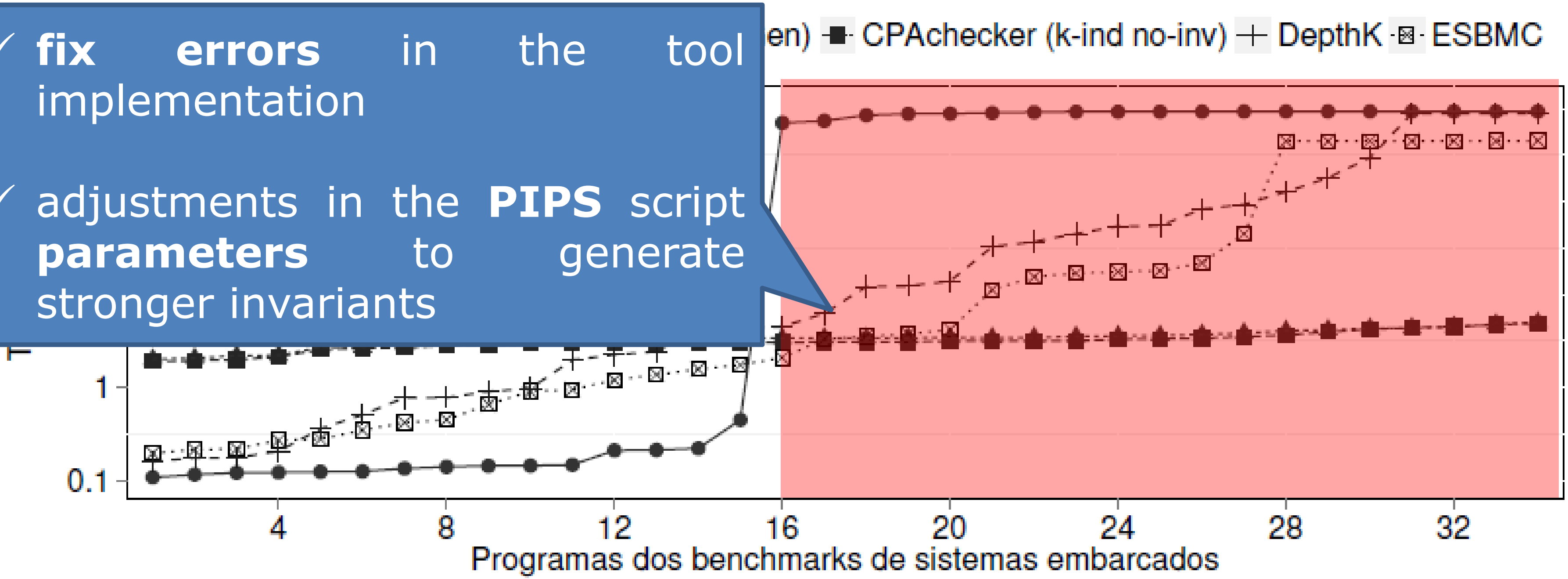


Experimental Results

The verification time:

✓ DepthK is **only faster** than CBMC

35.30% de Unknown



- ✓ **fix errors** in the tool implementation
- ✓ adjustments in the **PIPS** script **parameters** to generate stronger invariants

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Related Works

VS.

(Groβe, 2012)

- ✓ Explored proofs by mathematical induction of hardware and software systems
- ✓ Required changes in the code to introduce loop invariants

(Hagen et al., 2018 and Donaldson et al., 2011)

- ✓ The approach is similar
- ✓ Our method is completely automatic and does not require the user to provide loops invariants
- ✓ Not only as bug-finding tools
- ✓ DepthK aims prove correctness of C programs

Related Works

VS.

(Donaldson et al., 2010) - Scratch tool

- ✓ It is restricted to verify a specific class of problems for a particular type of hardware
- ✓ It requires annotations in the code to introduce loops invariants

(Beyer et al., 2015) - CPAChecker

- ✓ Invariant generation from predefined templates (**interval**) and constantly feeds the inductive step process
- ✓ DepthK adopts PIPS (polyhedral)

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Conclusions

The experimental results are promising:

- ✓ DepthK determined **11.27% more accurate results** than that obtained by **CPAChecker** in the SVCOMP 2015 loops subcategory
- ✓ DepthK determined **3.45% more correct results** to analyze all 176 C programs
- ✓ DepthK determined **17% more accurate results** than the *k*-induction algorithm without invariant

Conclusions

Improvements in the DepthK tool - In embedded systems benchmarks:

- ✓ DepthK only obtained better results than CBMC tool

For future works:

- ✓ We will improve the robustness of DepthK and tune the PIPS parameters to produce stronger invariants

Overview:

- ✓ In comparison to other state of the art tools, showed promising results indicating its effectiveness.

Questions ?

Thank you for your attention!

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