Counterexample Guided Inductive Optimization Applied to Mobile Robot Path Planning

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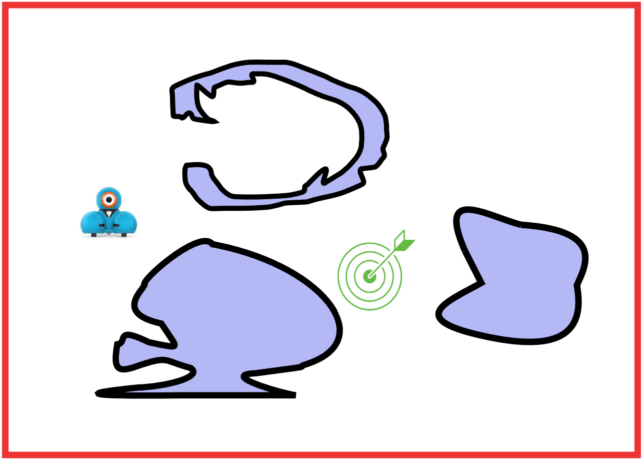
Motivation



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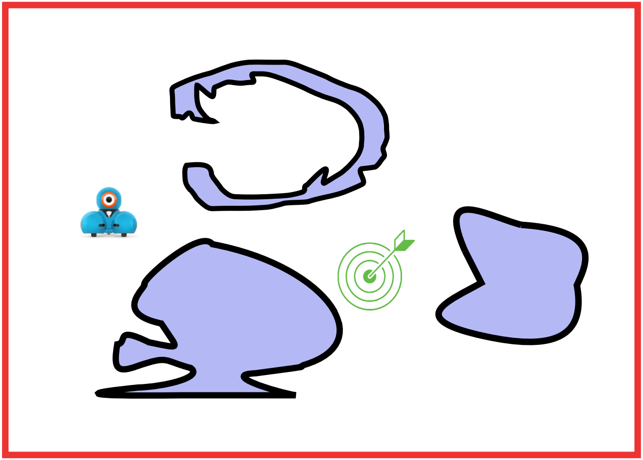

How to find a path from the starting point to the target point?

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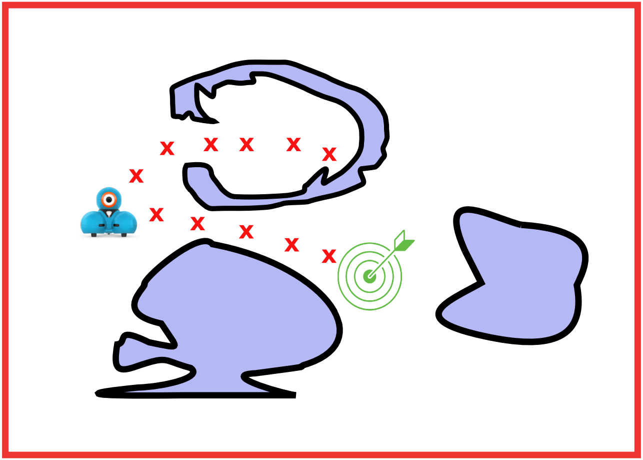

Path must meet safety constraints (e.g., obstacle avoidance)

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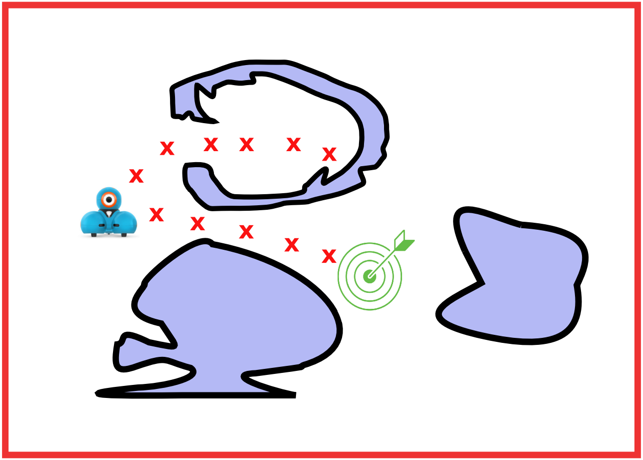

It is the objective of the **path planning** task

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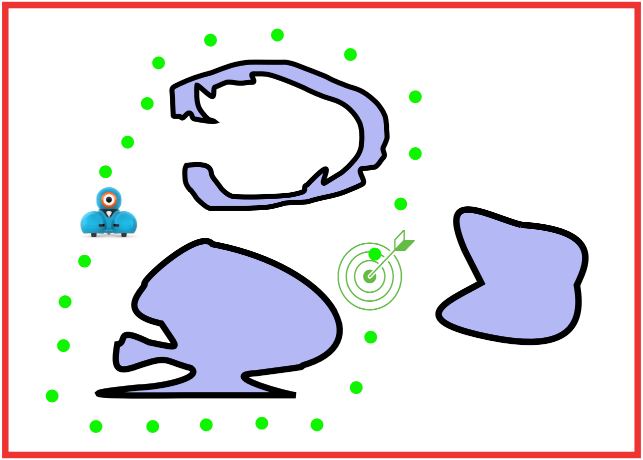

How to find the best path that meets the constraints?

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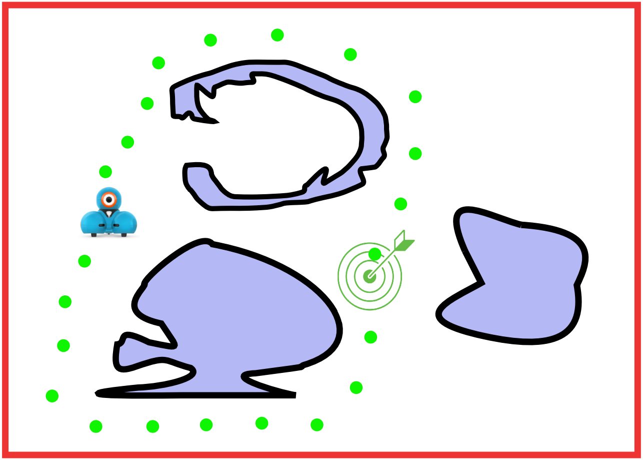

The path must be evaluated w.r.t. a cost function (e.g., distance and energy waste)

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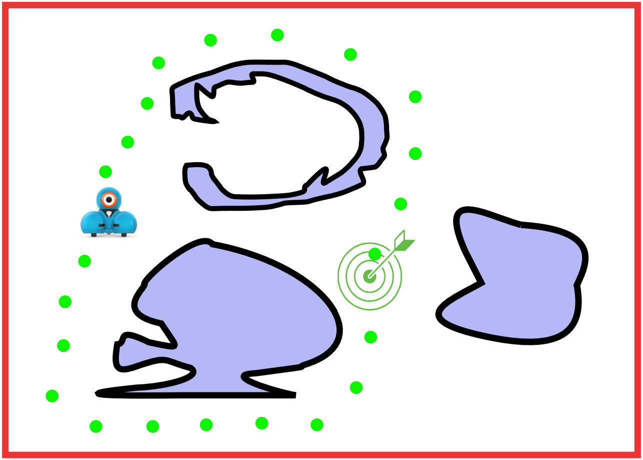

It is the objective of **optimal path planning**

Objectives

Apply Counterexample Guided Inductive Optimization (CEGIO) to mobile robot optimal path planning

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- Encode the environment, movement space, and static obstacles
- Parametrize the path by using the coordinates of path points and its respective orientations
- To find the shortest path that satisfies the constraints given by the problem

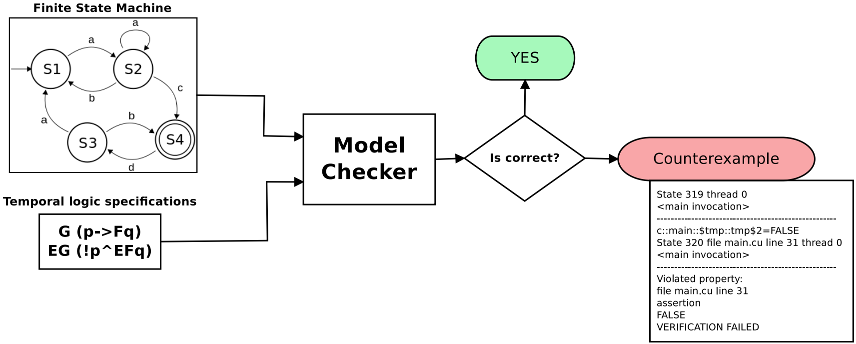
Model checking

Traditional path planning methodologies (e.g. APF- and GA-based algorithms) cannot ensure the global path optimality.

Model checking

CEGIO optimization ensures the global optimization because it is based on model checking procedures.

Model checking



Modeling the optimization problem using a model checker

The directives `ASSUME` and `ASSERT` must be employed for modeling optimization problems

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- ASSERT:is used for holding the global optimization condition $l_{optimal}$

$$l_{optimal} \leftrightarrow f(x) > f_p \quad (1)$$

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- Decision variables are defined as non-deterministic integers that represents rationals with desired precision
- The verification engine is executed by iteratively increasing the precision and converging to the optimal solution

CEGIO-based Path Planning

The main steps of CEGIO-based Path Planning are:

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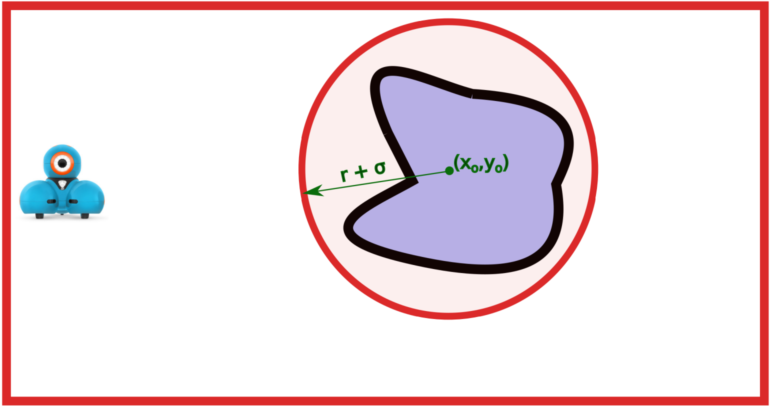
Step 2

Formulate the cost function

Step 3

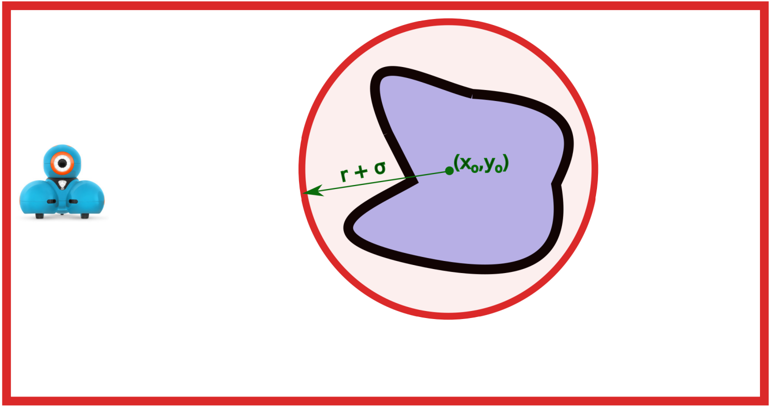
Parametrize the paths and find an optimal path that satisfies the constraints given by the problem

Environment Modeling



The search space is delimited by a rectangle

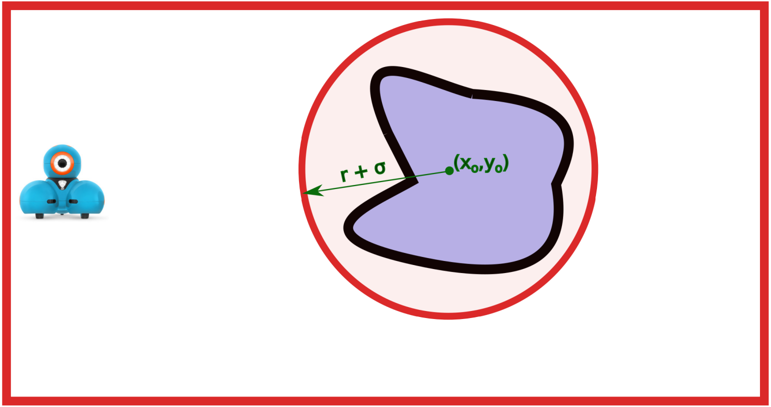
Environment Modeling



The obstacles are modeled as circles

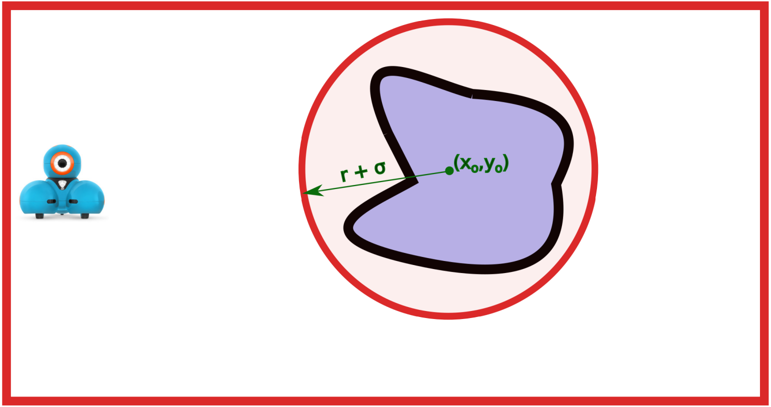
$$(x_{i\lambda} - x_0)^2 + (y_{i\lambda} - y_0)^2 \geq (r + \sigma)^2 \quad (2)$$

Environment Modeling



The constraints of the optimization problem must ensure that there is no intersection between the path and the obstacles

Path parametrization



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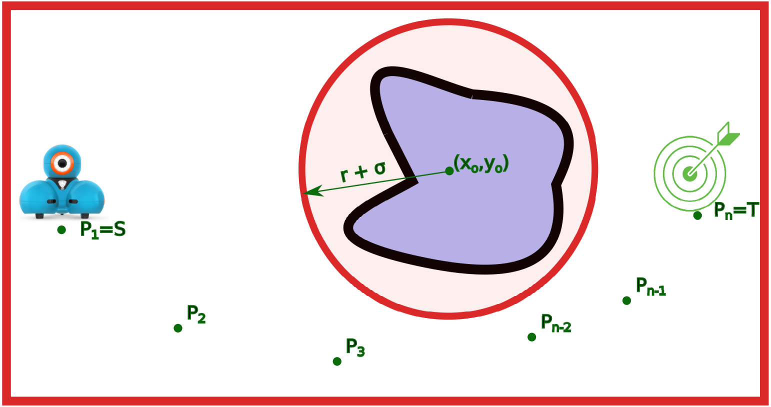
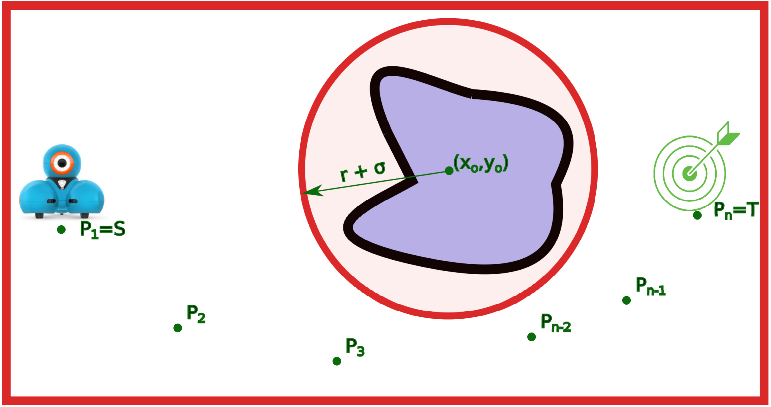


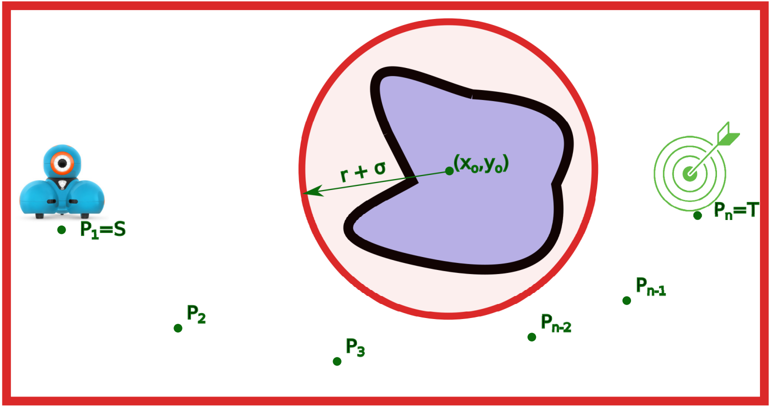
The bi-dimensional path has *n* vertices ($P_1, P_2, ..., P_n$)

Path parametrization



The path must start at the starting point $S = P_1$ and end at target point $T = P_n$

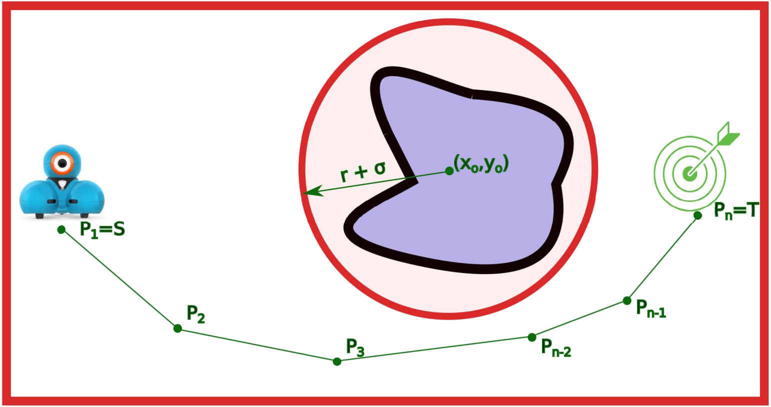
Path parametrization



The vertex matrix L is defined as follows.

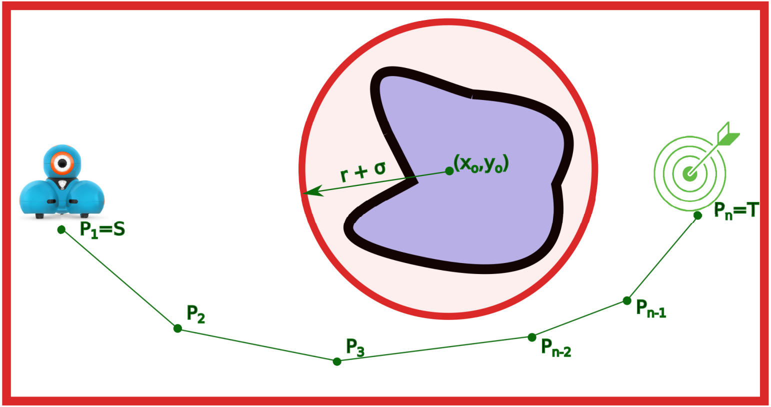
$$\mathbf{L} = [P_1, P_2, ..., P_{n-1}, P_n] \quad (2)$$

Path parametrization



The path is formed by $n-1$ straight segments. The i-th segment is built from P_i to P_{i+1}

Path parametrization



The set of points in the i-th segment is parametrized as follows for all $\lambda \in [0,1]$

$$p_{i\lambda}(L) = (1-\lambda)P_i + \lambda P_{i+1} \tag{2}$$

Path Optimization Problem

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$$J(L) = \sum_{i=1}^{n-1} \left\| P_{i+1} - P_i \right\|_2 , \tag{3}$$

The resulting optimization problem is:

$$\begin{aligned} \min_{L} \quad & J(L), \\ \text{s.t.} \quad & p_{i\lambda}(L) \notin \mathbb{O} \\ & p_{i\lambda}(L) \in \mathbb{E} \\ & i = 1, ..., n-1, \end{aligned} \tag{4}$$

The model checking procedure checks the satisfiability of $J_{optimal}$:

$$J_{optimal} \leftrightarrow J(L) > J_c \tag{5}$$

CEGIO-based path planning algorithm

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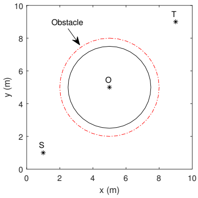
Input: Cost function $J(\mathbf{L})$, set of constraints Ω, desired precision η
Output: The optimal path and length ($\mathbf{L}^*$ and $J(\mathbf{L}^*)$)
Initialize $J(\mathbf{L}^{(0)})$ randomly, precision variable with $p = 1$, $k = 0$ e $i = 1$, number of points with $n = 1$
Declare $\mathbf{L}^i$ as non-deterministic integer vector
while $k \leq \eta$ **do**
    Find the best solution with the precision k
    $k = k + 1$
    Update the set Ω^k and the precision variable k
end while
$\mathbf{L}^* = \mathbf{L}^{(i)}$ and $J(\mathbf{L}^*) = J(\mathbf{L}^{(i)})$

CEGIO-based path planning algorithm

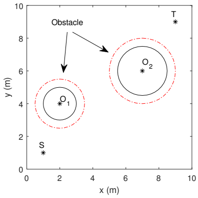
Find the best path with precision k

```
Define limits of L with directive ASSUME
Describe the objective function model J(L)
repeat
    ASSUME(J(L^(i)) < J(L^(i-1)))
    ASSERT(J_optimal)
    Update L* = L^(i) and J(L*) = J(L^(i)) based on the counterexample
    i=i+1;
until TRUE
if J_optimal is not consecutively SAT then
    Break
else
    Update n
end if
```

Benchmarks and Experimental Settings



(a) Environment 1



(b) Environment 2

Figure

Benchmarks and Experimental Settings

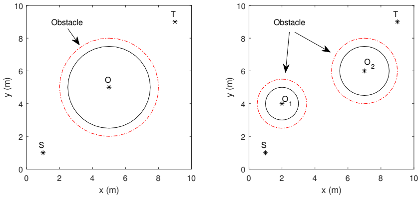


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Figure

- The experiments were performed in the two above environments

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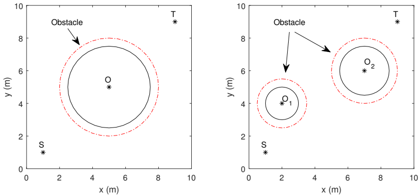

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- The experiments were performed in the two above environments
- All experiments were conducted on an otherwise idle Intel Core i7 4790 3.60 GHz processor, with 16 GB of RAM, running Ubuntu 14.10 64-bits

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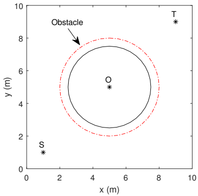

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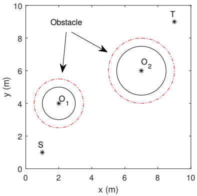

(b) Environment 2

Figure

- The experiments were performed in the two above environments
- All experiments were conducted on an otherwise idle Intel Core i7 4790 3.60 GHz processor, with 16 GB of RAM, running Ubuntu 14.10 64-bits
- The CBMC v4.5 with support to the MiniSAT v2.2.0 and ESBMC v3.1.0 with support to the MathSAT v5.3.13 were employed

Research Questions

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- **RQ1** Is it possible to apply CEGIO for robot mobile path planning?

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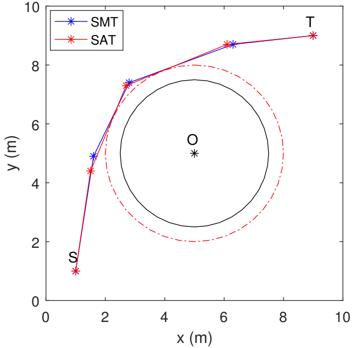
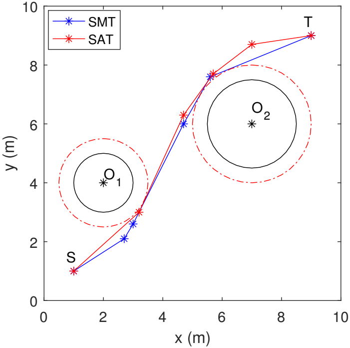
- **RQ2** Which CEGIO parameters can be adjusted to obtain a good trade off between planning time and cost?

Experimental Results



(a) Path for environment 1



(b) Path for environment 2

Figure

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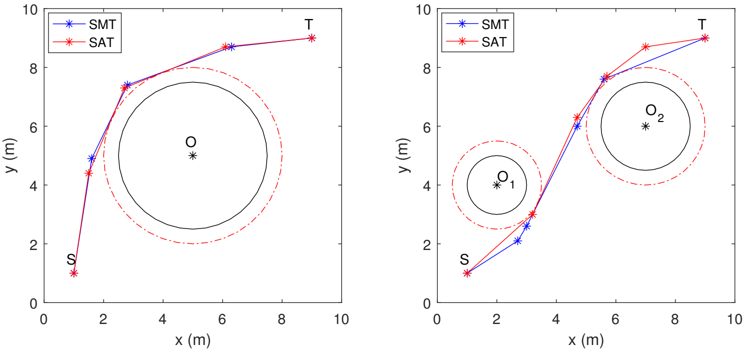

(a) Path for environment 1

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Figure

- Optimal paths are obtained for both settings with 5 and 6 points respectively

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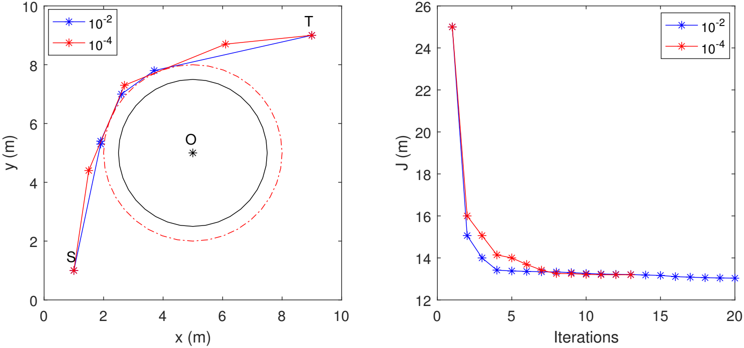

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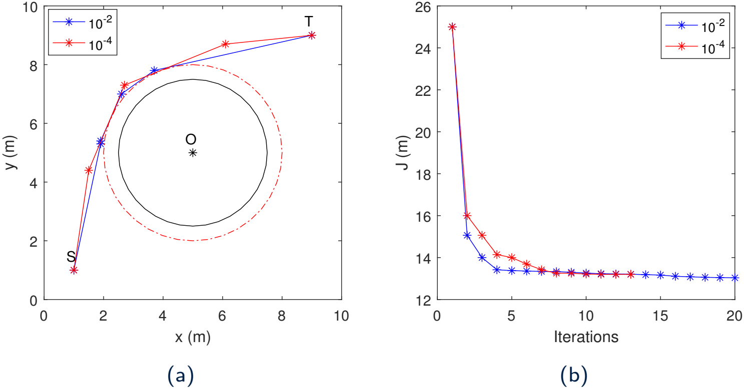

Figure

- The time spent is reduced to about 5% by reducing the precision from 10^{-4} to 10^{-2}

Conclusions

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- The time can be reduced by adjusting the precision and breaking the optimization process when it achieves the steady state
- Further studies include the usage of multi-objective optimization and applications to UAVs