Sound and Automated Synthesis of Digital Controllers for Continuous Plants

Alessandro Abate, Iury Bessa, Dario Cattaruzza, Lucas Cordeiro, Cristina David, Pascal Kesseli and Daniel Kroening

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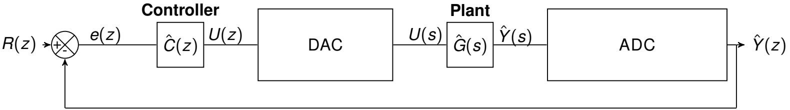
- digital control literature: success and limitations

- use of counterexample guided inductive synthesis (CEGIS) to automate design of sound digital controllers

CPS Setup: Continuous Plant and Digital Controller



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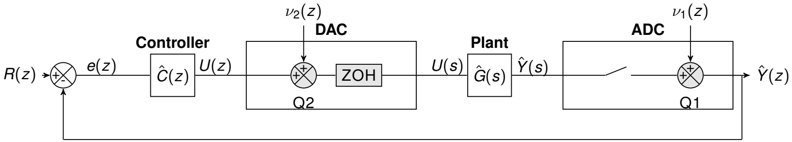

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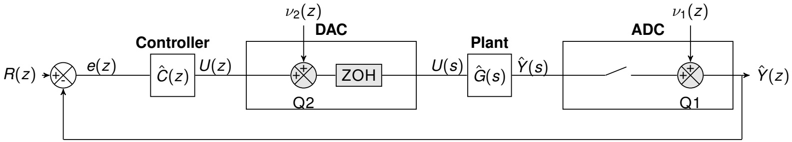

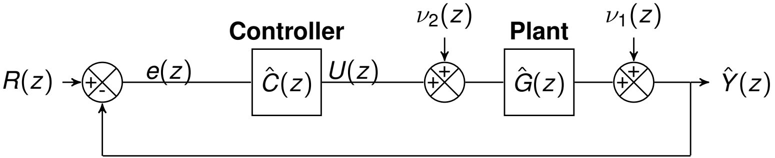

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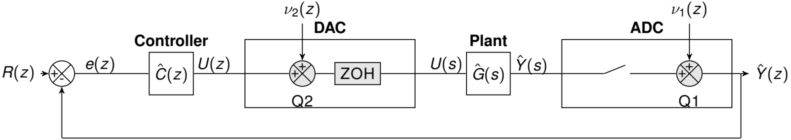

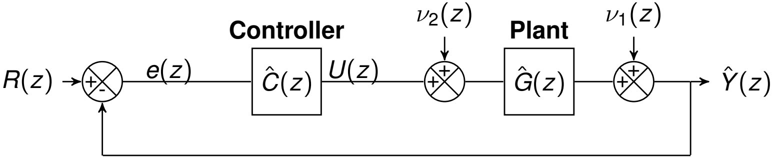

$$\hat{G}(z) = (1 - z^{-1})\mathcal{Z}\left\{\mathcal{L}^{-1}\left\{\frac{\hat{G}(s)}{s}\right\}_{t=kT}\right\}$$

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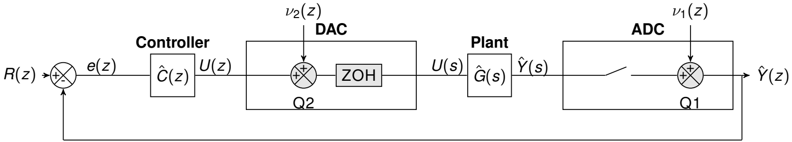

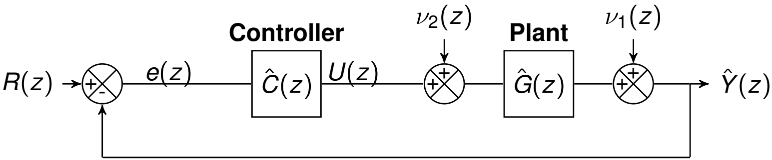

$$\hat{Y}(z) = \frac{\hat{G}(z)\hat{C}(z)}{1+\hat{G}(z)\hat{C}(z)}R(z) + \frac{1}{1+\hat{G}(z)\hat{C}(z)}\nu_1(z) + \frac{\hat{G}(z)}{1+\hat{G}(z)\hat{C}(z)}\nu_2(z)$$

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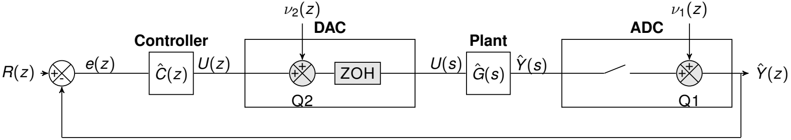

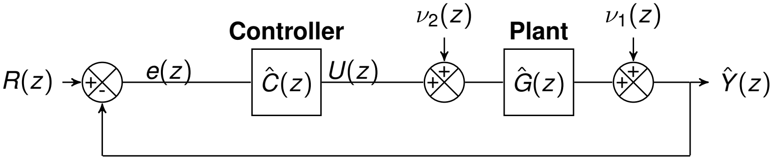

Common denominator:

$$\hat{G}_n(z)\hat{C}_n(z) + \hat{G}_d(z)\hat{C}_d(z)$$

Stability Analysis via Jury's Criterion

$$S(z) = \hat{G}_n(z)\hat{C}_n(z) + \hat{G}_d(z)\hat{C}_d(z) = \sum_{i=0}^{N_S} c_i z^{N_s - i}, c_0 \neq 0$$

$$M = \begin{pmatrix} v_1^{(0)} & \cdots & v_{N_S}^{(0)} \\ v_1^{(1)} & \cdots & v_{N_S}^{(1)} \\ \vdots & \vdots & \vdots \\ v_1^{(N_S-1)} & \cdots & v_{N_S}^{(N_S-1)} \end{pmatrix},$$

$$v_j^{(k)} = \begin{cases} c_{j-1}, & k = 0 \\ 0, & k > 0 \wedge \text{if } j > N_S - k \\ v_1^{(k-1)} - v_{N_S-j}^{(k-1)} \cdot \frac{v_1^{(k-1)}}{v_{N_S-k}^{(k-1)}}, & k > 0 \wedge \text{if } j \leq N_S - k \end{cases}$$

$$S(z) \text{ is stable } \Leftrightarrow$$

$S(1) > 0$

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Transfer Functions and Uncertainty

$$\hat{C}(z) = \frac{\hat{C}_n(z)}{\hat{C}_d(z)} = \frac{\sum_{i=0}^{M_C}(\beta_i + \Delta\beta_i)z^{-i}}{\sum_{i=0}^{N_C}(\alpha_i + \Delta\alpha_i)z^{-i}},$$

$$\hat{G}(z) = \frac{\hat{G}_n(z)}{\hat{G}_d(z)} = \frac{\sum_{i=0}^{M_G}(b_i + \Delta b_i)z^{-i}}{\sum_{i=0}^{N_G}(a_i + \Delta a_i)z^{-i}}.$$

- $\vec{\beta}$ and $\vec{\alpha}$ – vectors containing controller coefficients
- $\vec{b}$ and $\vec{a}$ – plant coefficients

Sources of Uncertainty

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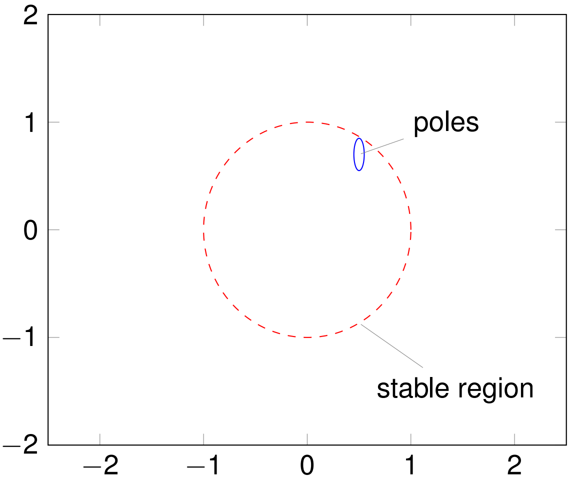
3. Roundoff errors in verification process (affect $\Delta S(z)$)
4. Discretisation errors (affect $\Delta G(z)$ and $\Delta C(z)$)

Pole Placement



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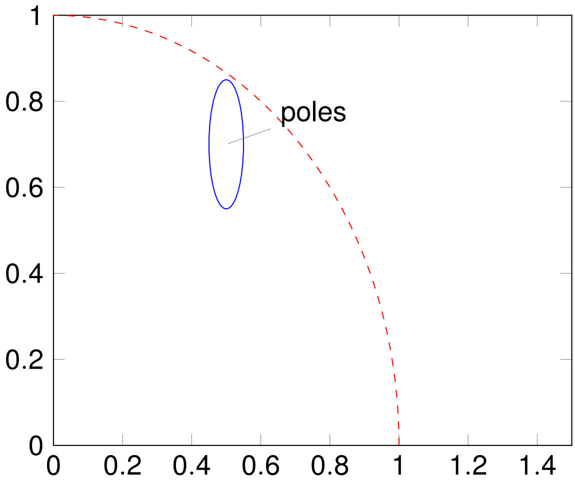
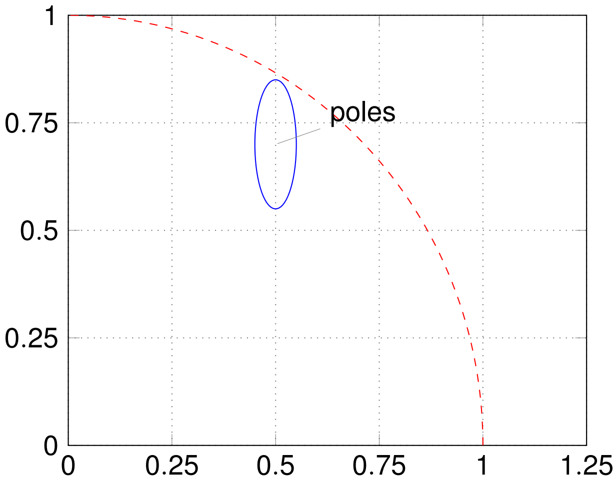


Pole Placement in a Discretized Domain



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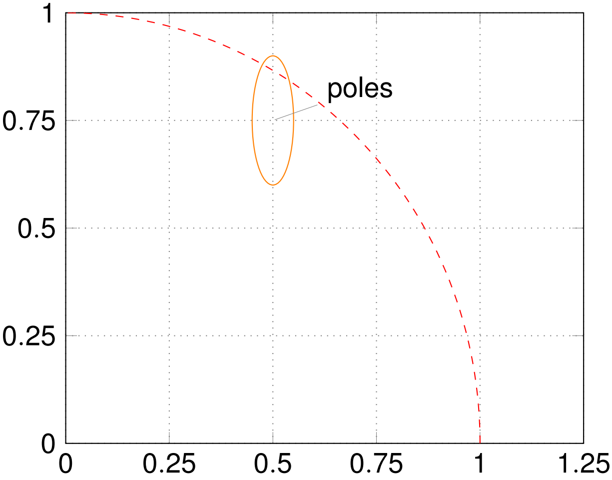
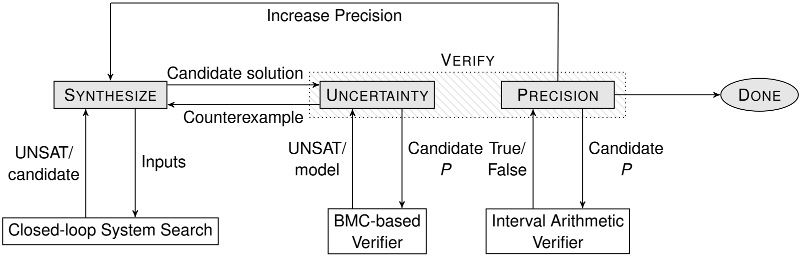


Objective

We seek to synthesise a controller where $\Delta\vec{\beta} = 0$ and $\Delta\vec{\alpha} = 0$ and which stabilizes all plants within a given range of $\Delta\vec{b}$ and $\Delta\vec{a}$.



Case Study

Take a classical cruise control example from the literature.

$$G(z) = \frac{0.0264}{z - 0.9998}.$$

Using an optimization tool, the following stable high-performance controller was designed:

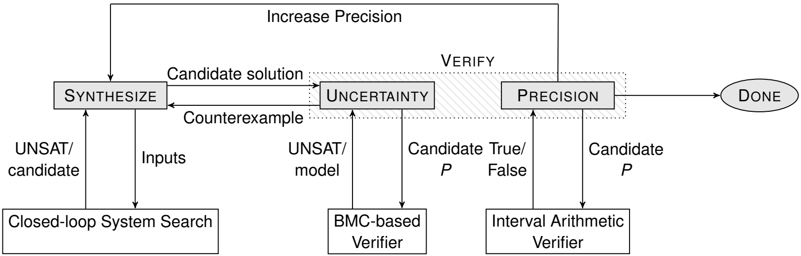
$$C(z) = \frac{2.72z^2 - 4.153z + 1.896}{z^2 - 1.844z + 0.8496}.$$

However, if we implement the controller in $\mathbb{R}\langle 4, 16\rangle$.

$$\tilde{C}(z) := \frac{2.7199859619140625z^2 - 4.1529998779296875z + 1.89599609375}{z^2 - 1.843994140625z + 0.8495941162109375}.$$

where $\tilde{C}(z)$ is the controller adjusted to the FWL. The resulting system is unstable.

CEGIS Architecture



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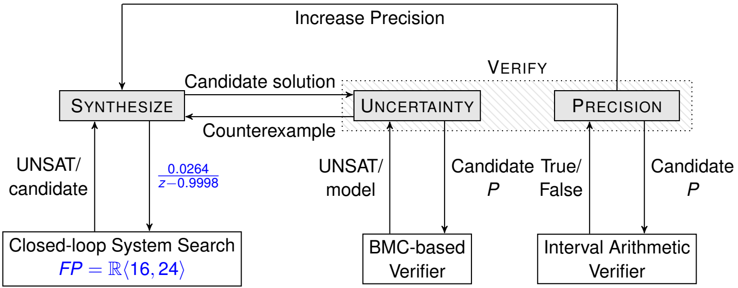

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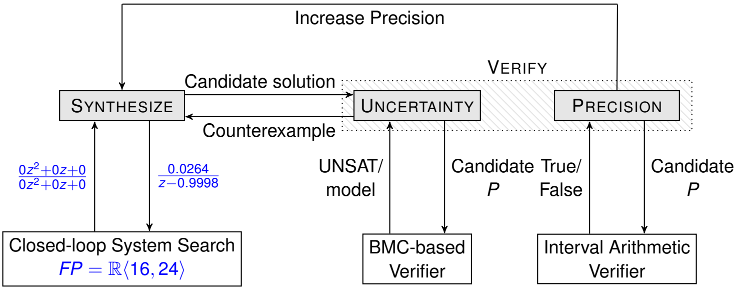

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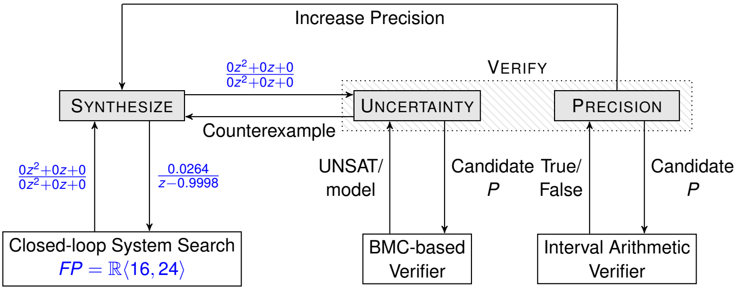

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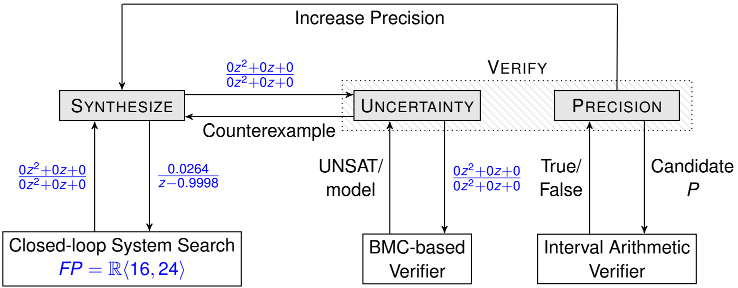

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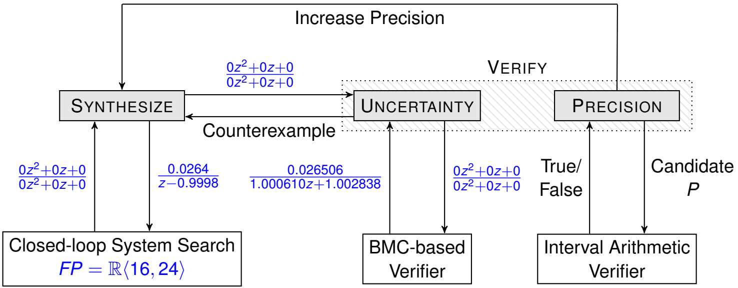

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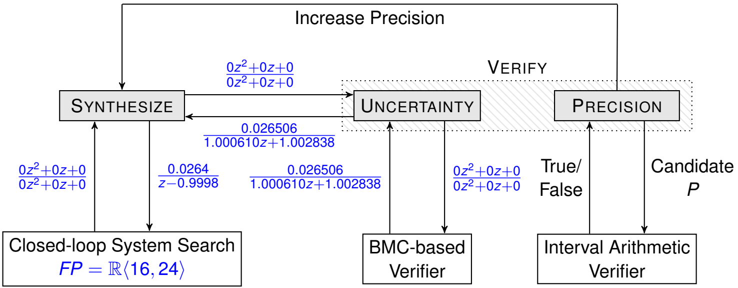

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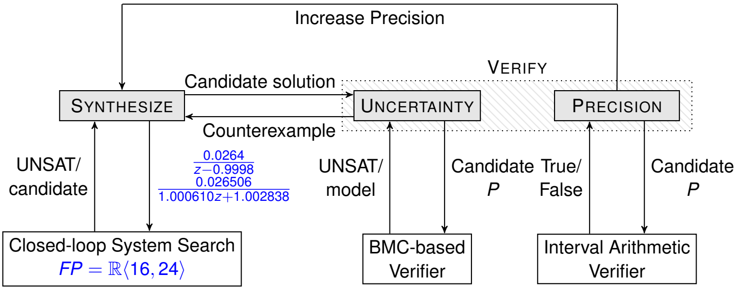

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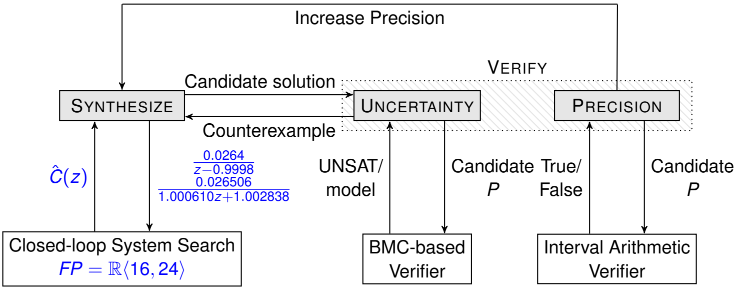

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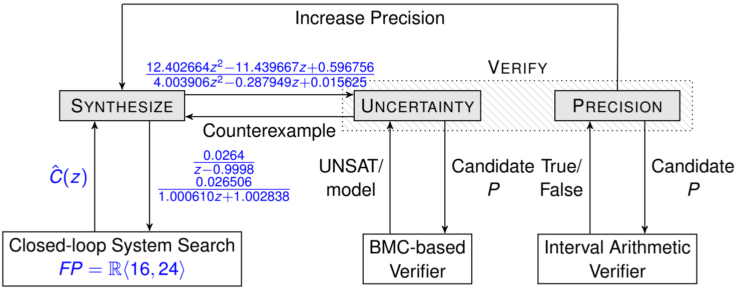

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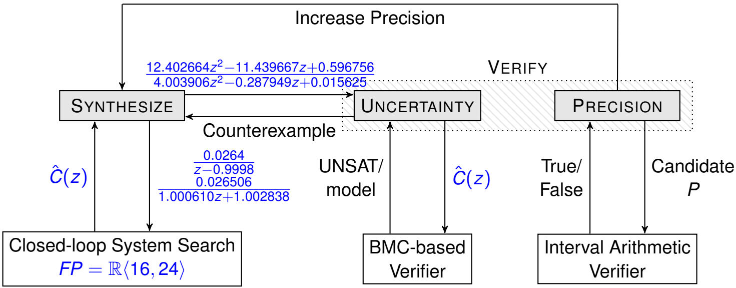

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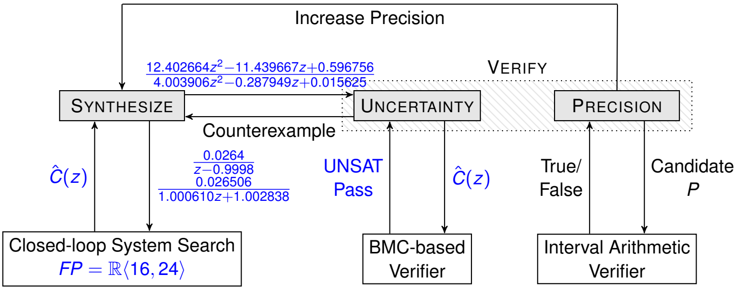

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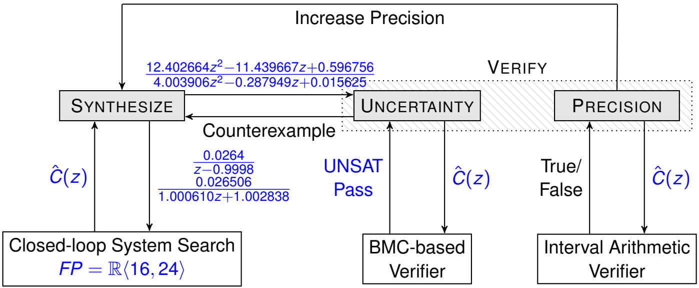

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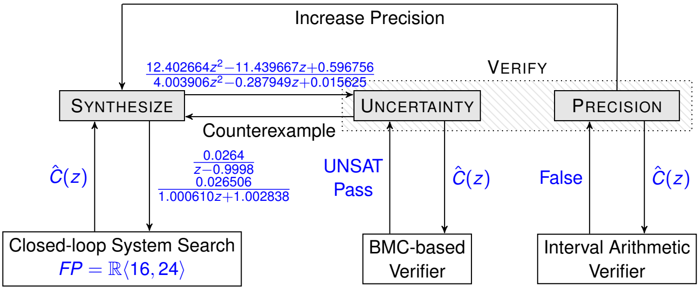

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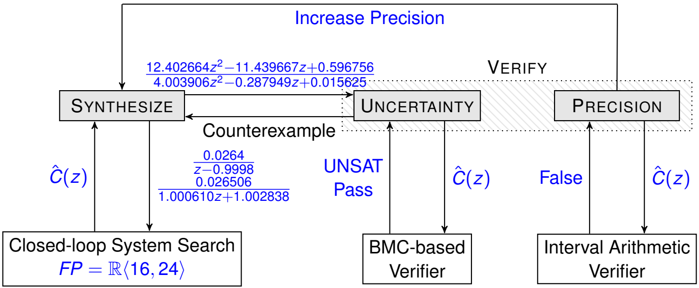

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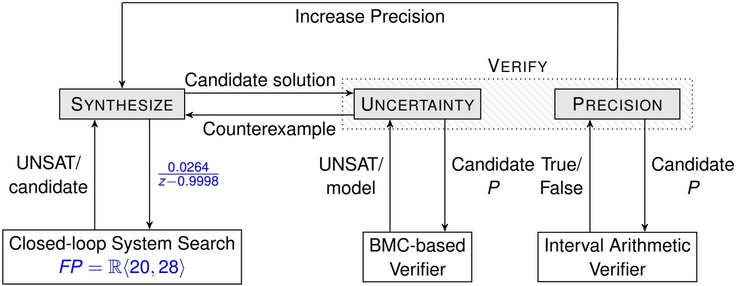

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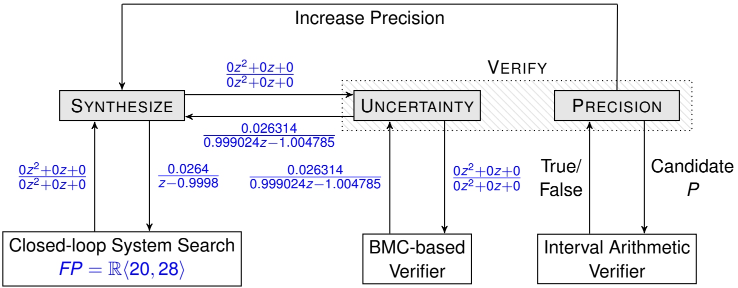

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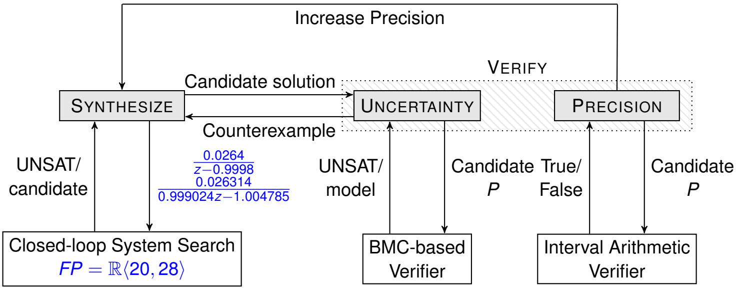

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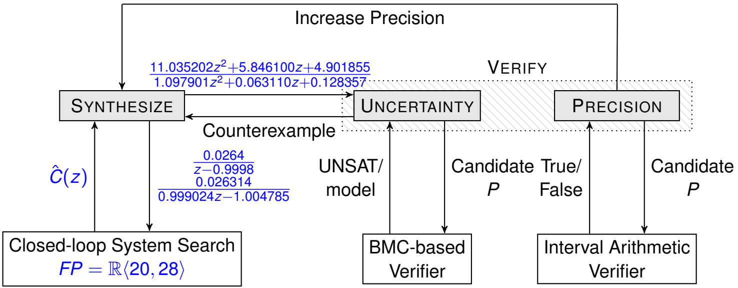

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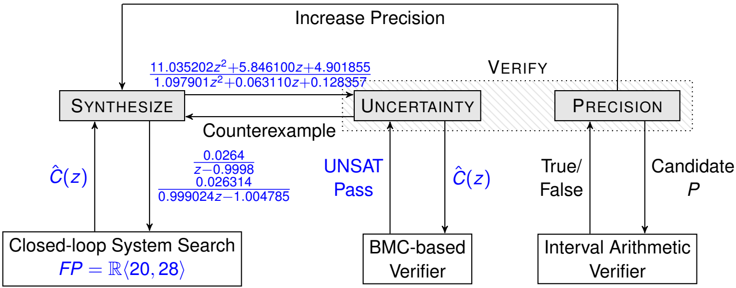

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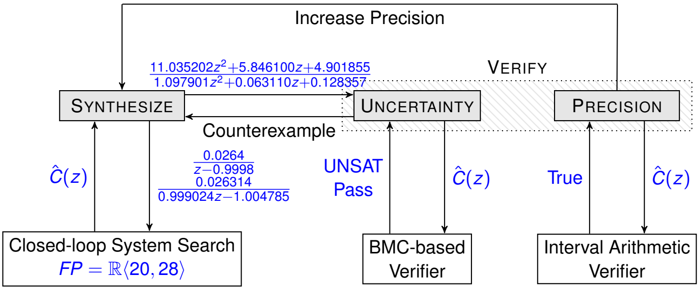

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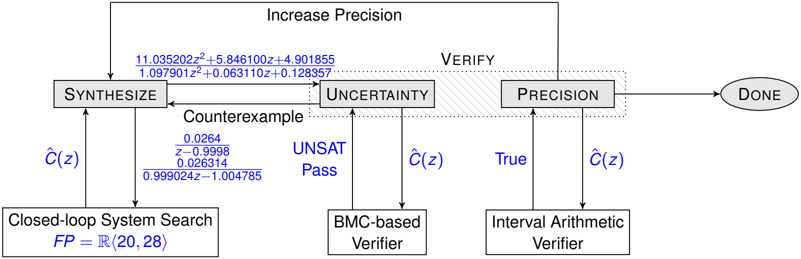
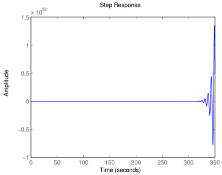
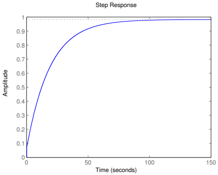


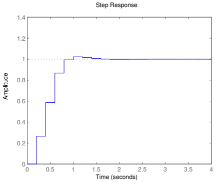
Outcomes: Step Response



(a) Original controller



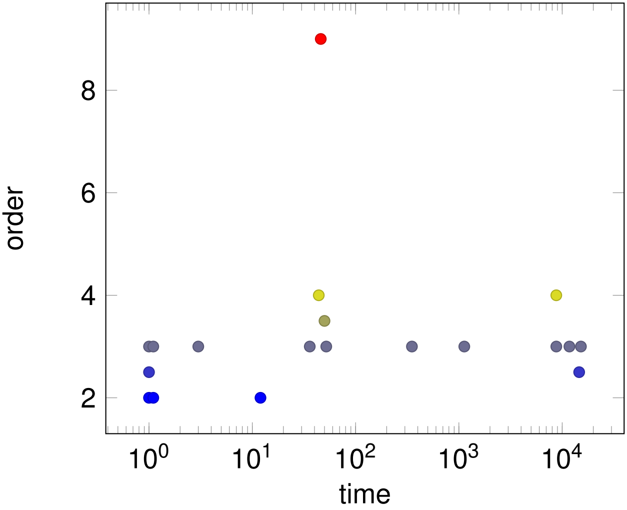
(b) Controller (first step)



(c) Controller (final step)

- original controller was synthesised ignoring FWL effects (becomes unstable over time)

Outcomes: Controller Order



Outcomes: Benchmark

#	Plant	Benchmark	I	F	2-stage	1-stage
1	G_{1a}	CruiseControl02	4	16	**12 s**	67 s
2	G_{1b}	CruiseControl02†	4	16	14600 s	**52 s**
3	G_{2a}	SpgMsDamper	15	16	**52 s**	318 s
4	G_{2b}	SpgMsDamper†	15	16	✗	✗
5	G_{3a}	SatelliteB2	3	7	**36 s**	✗
6	G_{3b}	SatelliteB2†	3	7	✗	**4111 s**
7	G_{3c}	SatelliteC2	3	5	**3 s**	205 s
8	G_{3d}	SatelliteC2†	3	5	**50 s**	1315 s
9	G_4	Cruise	3	7	1 s	1 s
10	G_5	DCMotor	3	7	**1 s**	10 s
11	G_6	DCServomotor	4	11	**46 s**	✗
12	G_7	Doyleetal	4	11	**8769 s**	✗
13	G_8	Helicopter	3	7	**44 s**	✗
14	G_9	Pendulum	3	7	**1 s**	14826 s
15	G_{10}	Suspension	3	7	**1 s**	5 s
16	G_{11}	Tapedriver	3	7	1 s	1 s
17	G_{12a}	a_ST1_IMPL1	16	4	**11748 s**	✗
18	G_{12a}	a_ST1_IMPL2	16	8	**351 s**	✗
19	G_{12a}	a_ST1_IMPL3	16	12	**8772 s**	✗
20	G_{12b}	a_ST2_IMPL1	16	4	**1128 s**	✗
21	G_{12b}	a_ST2_IMPL2	16	8	✗	✗
22	G_{12b}	a_ST2_IMPL3	16	12	**15183 s**	✗
23	G_{12c}	a_ST3_IMPL1	16	4	✗	✗

Table: DSSynth results (✗ = time-out, † = uncertainty)

Conclusions

- We have presented a method for automatically synthesizing stable and sound controllers, implemented in a tool called DSSynth.
- DSSynth marks the first use of the CEGIS that handles plants with uncertain models and FWL effects over the digital controller.
- Our experimental results show that DSSynth is able to synthesize stable controllers for most benchmarks within a reasonable amount of time.
- Future work will extend this CEGIS-based approach to LTI systems with state space safety specifications.