DSSynth: An Automated Digital Controller Synthesis Tool for Physical Plants
ASE 2017

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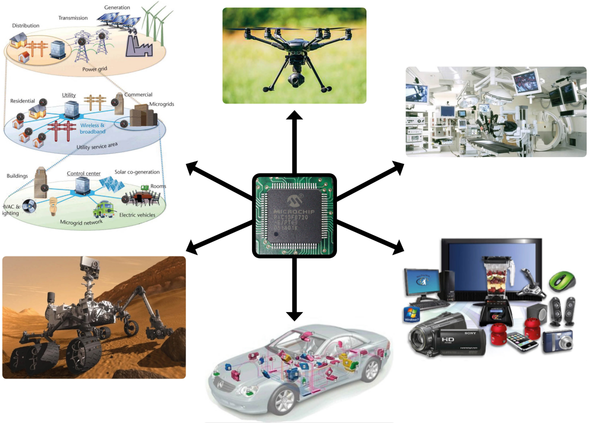
Motivation



Generation
Transmission
Distribution
Power grid
Residential
Utility
Commercial
Microgrids
Wireless & broadband
Utility service area
Buildings
Control center
Solar co-generation
Rooms
Electric vehicles
Microgrid network
MICROCHIP

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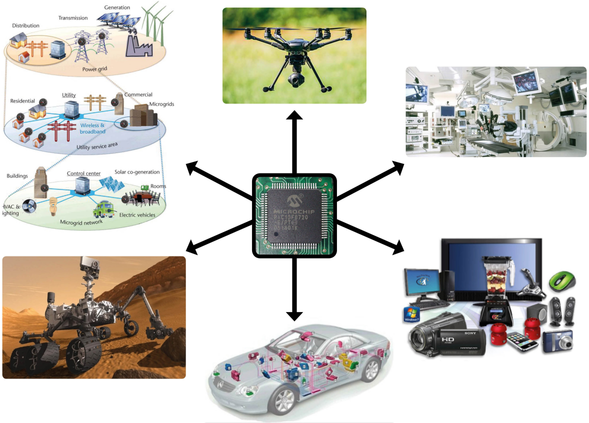
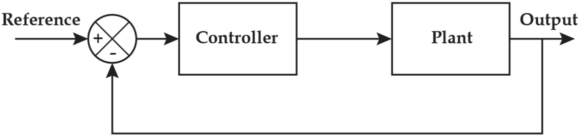


Automatically synthesise digital controllers

Typical closed-loop control system



- Representation of the digital controller and plant
  - state-space: matrices A, B, C, and D
  - transfer-function: coefficients b_0, b_1,...,b_m and a_0, a_1,...,a_m

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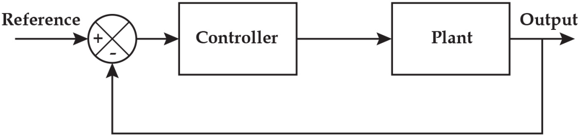

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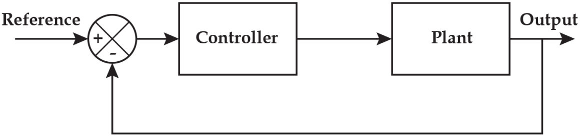

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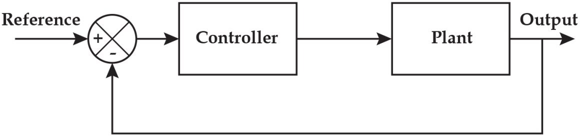

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- Numerical erros (truncation and rounding)

Objectives

Generate sound digital controllers for stability and safety specifications with a very high degree of automation

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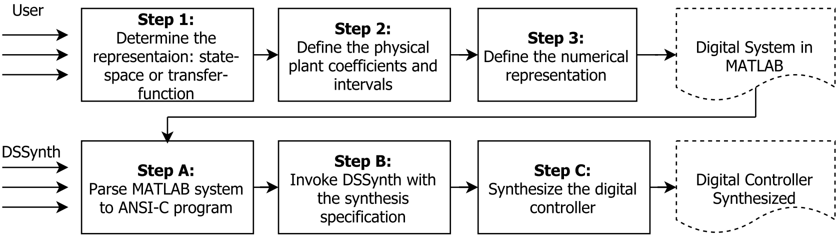
- support for transfer-function and state-space representations in closed-loop form
- synthesize different numerical representations of the controller using CounterExample Guided Inductive Synthesis (CEGIS)
- provide a MATLAB toolbox to synthesize digital controllers while taking into account finite word-length effects

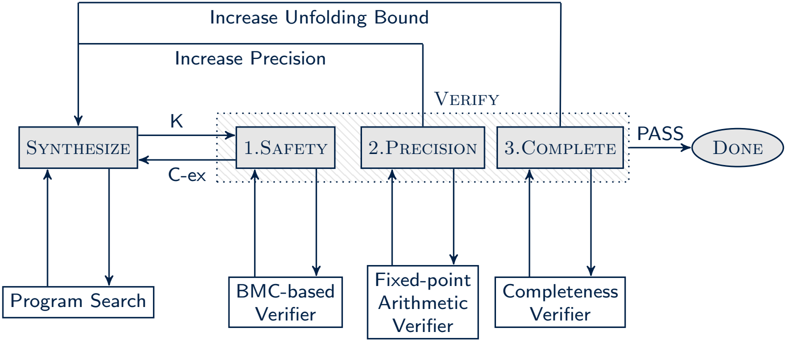
The Proposed Synthesis Methodology

Phases of the controller synthesis:



CEGIS for Control Systems

CEGIS with multi-staged verification:



DSSynth Usage - Transfer Function

Physical plant for an unmanned aerial vehicle (UAV) plant:

$$G(z) = \frac{B(z)}{A(z)} = \frac{-0.06875z^2}{z^2 - 1.696z + 0.7089}. \quad (1)$$

Synthesizing the digital controller:

```
>> num = [-0.06875 0 0];
>> den = [1.0000 -1.696 0.7089];
>> system = tf(num,den,0.002);
>> y = synthesize(system,8,8,1,-1);
>> SYNTHESIS SUCCESSFUL
>> y =
>>   -0.9983z^2 + 0.09587z + 0.1926
>>   --------------------------------
>>        z^2 + 0.5665z + 0.75
```

DSSynth Usage - Transfer Function

Digital controller synthesized by DSSynth:

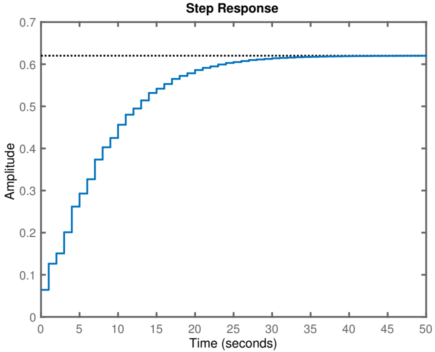
$$C(z) = \frac{-0.9983^2 + 0.09587z + 0.1926}{z^2 + 0.5665z + 0.75}. \quad (2)$$

Computing the general equation (plant and controller):

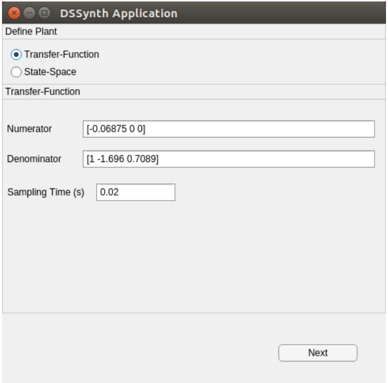
```
>> num = [-0.99832 0.09587 0.1926];
>> den = [1 0.5665 0.75];
>> controller = tf(num,den,0.002);
>> num = [-0.06875 0 0];
>> den = [1.0000 -1.696 0.7089];
>> plant = tf(num,den,0.002);
>> sys = feedback(series(controller, plant),1)
>> sys =
>>       0.06863z^4 - 0.006591z^3 - 0.01324z^2
>> ---------------------------------------------
>> 1.069z^4 - 1.136z^3 + 0.4849z^2 - 0.8704z + 0.5317
```

DSSynth Usage - Step Response

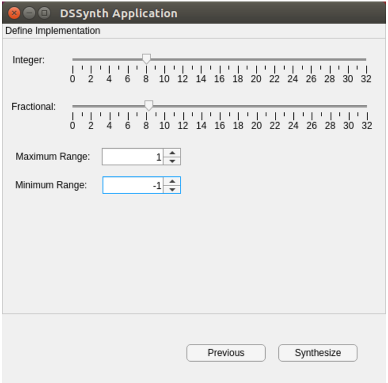
Step response for the UAV plant describing a stable system:



DSSynth Usage - MATLAB Application

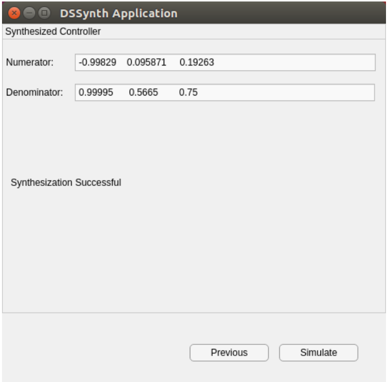


(a) Definition of the system representation and the physical plant

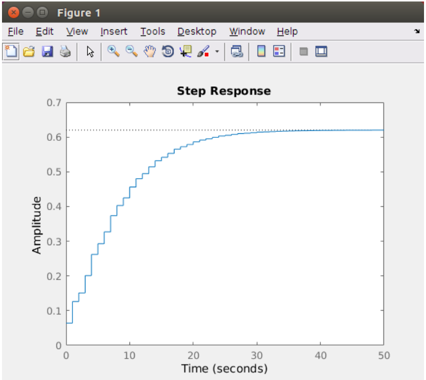


(b) Definition of implementation aspects and input ranges

DSSynth Usage - MATLAB Application



(c) Digital controller synthesized by DSSynth



(d) Step response for the synthesized digital controller

Experimental Evaluation

Our evaluation consists of 18 Single-Input and Single-Output control system benchmarks extracted from the literature:

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Experimental Objectives:

- Evaluate the DSSynth performance to produce digital controllers
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Experimental Objectives:

- Evaluate the DSSynth performance to produce digital controllers
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Experimental Setup:

- Signal input range: $\langle -1, 1 \rangle$
- Implementation features: $\langle 8, 8 \rangle$
- Intel Core $\text{i7} - 2600$ 3.40 GHz processor with 24 GB of RAM

Experimental Evaluation

Experimental Results:

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DSSynth Matlab toolbox:
https://www.cprover.org/DSSynth/dssynth-toolbox-1.0.0.zip
https://github.com/ssvlab/dsverifier/tree/master/toolbox-dssynth