

Safe, Automated and Formal Synthesis of Digital Controllers for Continuous Plants

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We present a sound and automated approach to synthesizing safe, digital controllers for physical plants represented as linear, time-invariant models. The synthesis accounts for errors caused by the digitization effects introduced by digital controllers operating in either fixed- or floating-point arithmetic. Our approach uses counterexample-guided inductive synthesis (CEGIS): in the first phase an inductive generalisation engine produces a possible solution that is safe for some possible initial conditions but may not be safe for all initial conditions. Safety for all initial conditions is then verified in a second phase either via bounded model checking or abstract acceleration; if the verification step fails, a counterexample is provided to the inductive generalisation and the process iterates until a safe controller is obtained. We implemented our approach in a tool named DSSynth (Digital-System Synthesizer) and demonstrate its practical value by automatically synthesizing safe controllers for physical plant models from the digital control literature.

1 Introduction

Embedded control systems using fixed and floating-point arithmetic have become widespread as the availability of low-cost devices that can perform highly non-trivial control tasks has increased. Correct synthesis of control software for such platforms is non-trivial due to the digital representation of continuous quantities used by the controller. This representation introduces errors due to finite-precision arithmetic, time discretization and A/D - D/A conversions. Given an LTI model of a physical (continuous) plant, we present two automated approaches for generating correct-by-construction digital controllers that address all these challenges and satisfy a safety property for the physical plant. Both approaches make use of CounterExample-Guided Inductive Synthesis (CEGIS) [4, 6]. CEGIS is an iterative process, where each iteration performs inductive generalisation based on counterexamples provided by a verification module. Our two instantiations of CEGIS are described next.

The first approach starts by devising a digital controller that stabilizes the system’s model, while remaining safe for a pre-selected time horizon and a single initial state; then, it verifies unbounded-time safety by unfolding the dynamics of the LTI model, considering the full set of initial states, and checking a completeness threshold [5]. *The second approach* employs *abstract acceleration* [3] to evaluate all possible progressions of the LTI model simultaneously. This approach uses *abstraction refinement*, enabling us to start with a simple description regardless of the dynamics complexity, and only expand to more complex models when a solution cannot be found.

We provide experimental results showing that both our approaches are able to synthesize safe controllers for a set of physical plant models taken from digital control literature.

This abstract contains material published in [1, 2].

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