



Detection of Software Vulnerabilities: Static Analysis (Part I)

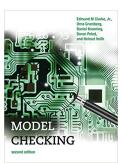
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Static Analysis

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 - Office: 2.28
 - Office hours: 15-16 Tuesday, 14-15 Wednesday
- Textbook:
 - Model checking (Chapter 14)
 - Software model checking. ACM Comput. Surv., 2009
 - The Cyber Security Body of Knowledge, 2019
 - Software Engineering (Chapters 8, 13)





- Functionality demanded increased significantly
 - Peer reviewing and testing

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```
void *threadA(void *arg) {
  lock(&mutex);
  x++;
  if (x == 1) lock(&lock);
  unlock(&mutex);
  lock(&mutex);
  x--;
  if (x == 0) unlock(&lock);
  unlock(&mutex);
}
```

```
void *threadB(void *arg) {
  lock(&mutex);
  y++;
  if (y == 1) lock(&lock);
  unlock(&mutex);
  lock(&mutex);
  y--;
  if (y == 0) unlock(&lock);
  unlock(&mutex);
}
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```
void *threadA(void *arg) {
  lock(&mutex);
  x++;
  if (x == 1) lock(&lock);
  unlock(&mutex); (CS1)
  lock(&mutex);
  x--;
  if (x == 0) unlock(&lock);
  unlock(&mutex);
}
```

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void *threadB(void *arg) {
  lock(&mutex);
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  lock(&mutex);
  x++;
  if (x == 1) lock(&lock);
  unlock(&mutex); (CS1)
  lock(&mutex);
  x--;
  if (x == 0) unlock(&lock);
  unlock(&mutex);
}
```

```
void *threadB(void *arg) {
  lock(&mutex);
  y++;
  if (y == 1) lock(&lock); (CS2)
  unlock(&mutex);
  lock(&mutex);
  y--;
  if (y == 0) unlock(&lock);
  unlock(&mutex);
}
```

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```
void *threadA(void *arg) {
  lock(&mutex);
  x++;
  if (x == 1) lock(&lock);
  unlock(&mutex); (CS1)
  lock(&mutex); (CS3)
  x--;
  if (x == 0) unlock(&lock);
  unlock(&mutex);
}
```

```
void *threadB(void *arg) {
  lock(&mutex);
  y++;
  if (y == 1) lock(&lock); (CS2)
  unlock(&mutex);
  lock(&mutex);
  y--;
  if (y == 0) unlock(&lock);
  unlock(&mutex);
}
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                                          void *threadB(void *arg) {
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                                           lock(&mutex);
 X++;
 if (x == 1) lock(\&lock):
                                              ==1) lock(&lock); (CS2)
                                 Deadlock <a href="https://ckenutex">Deadlock</a> <a href="https://ckenutex">Ck(&mutex);</a>
 unlock(&mutex); (CS1)
                                             ck(&mutex);
 lock(&mutex);
 X--;
 if (x == 0) unlock(&lock);
                                           if (y == 0) unlock(&lock);
 unlock(&mutex);
                                           unlock(&mutex);
```

Introduce software verification and validation

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- Understand soundness and completeness concerning detection techniques

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 - The software should conform to its specification
- Validation: "Are we building the right product?"
 - The software should do what the user requires
- Verification and validation must be applied at each stage in the software process
 - The discovery of defects in a system
 - The assessment of whether or not the system is usable in an operational situation

Static and Dynamic Verification

- Software inspections are concerned with the analysis of the static system representation to discover problems (static verification)
 - Supplement by tool-based document and code analysis
 - Code analysis can prove the absence of errors but might subject to incorrect results

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- Software inspections are concerned with the analysis of the static system representation to discover problems (static verification)
 - Supplement by tool-based document and code analysis
 - Code analysis can prove the absence of errors but might subject to incorrect results
- Software testing is concerned with exercising and observing product behaviour (dynamic verification)
 - The system is executed with test data
 - Operational behaviour is observed
 - Can reveal the presence of errors NOT their absence

Static and Dynamic Verification



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V & V planning

- Careful planning is required to get the most out of dynamic and static verification
 - Planning should start early in the development process
 - The plan should identify the balance between static and dynamic verification

V & V planning

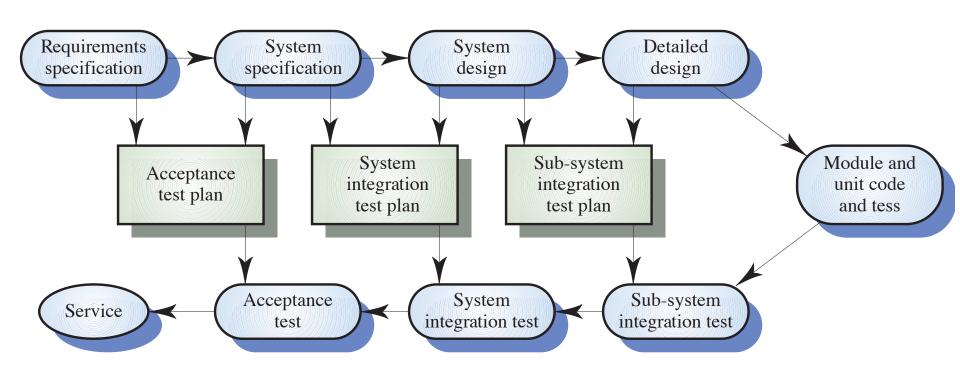
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V & V planning depends on system's purpose, user expectations and marketing environment

The V-model of development



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- Trade-off between soundness and completeness
 - A detection technique is sound for a given category if it can correctly conclude that a given program has no vulnerabilities
 - o An unsound detection technique may have *false negatives*, i.e., actual vulnerabilities that the detection technique fails to find
 - A detection technique is complete for a given category, if any vulnerability it finds is an actual vulnerability
 - o An incomplete detection technique may have *false positives*, i.e., it may detect issues that do not turn out to be actual vulnerabilities

- Achieving soundness requires reasoning about all executions of a program (usually an infinite number)
 - This can be done by static checking of the program code while making suitable abstractions of the executions

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 - This can be done by static checking of the program code while making suitable abstractions of the executions
- Achieving completeness can be done by performing actual, concrete executions of a program that are witnesses to any vulnerability reported
 - The analysis technique has to come up with concrete inputs for the program that triggers a vulnerability
 - A typical dynamic approach is software testing: the tester writes test cases with concrete inputs and specific checks for the outputs

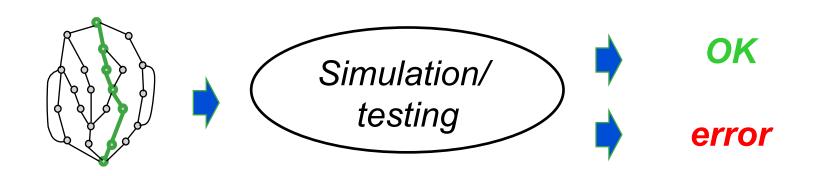
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Dynamic verification should be used in conjunction with **static verification** to provide **full code coverage**

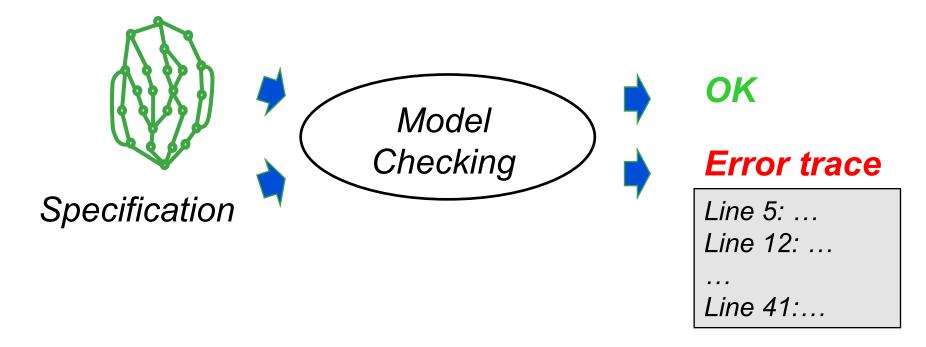
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Static analysis vs Testing/ Simulation



- Checks only some of the system executions
 - May miss errors
- A successful execution is an execution that discovers one or more errors

Static analysis vs Testing/ Simulation



- Exhaustively explores all executions
- Report errors as traces
- May produce incorrect results

Avoiding state space explosion

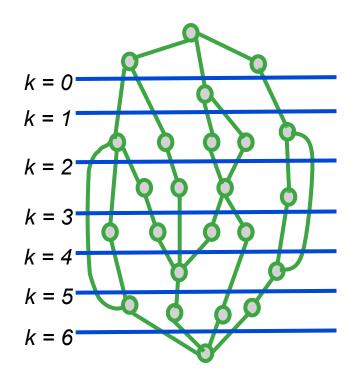
- Bounded Model Checking (BMC)
 - Breadth-first search (BFS) approach

- Symbolic Execution
 - Depth-first search (DFS) approach

Bounded Model Checking

A graph G = (V, E) consists of:

- V: a set of vertices or nodes
- E ⊆ V x V: set of edges connecting the nodes



- Bounded model checkers explore the state space in depth
- Can only prove correctness if all states are reachable within the bound

Breadth-First Search (BFS)

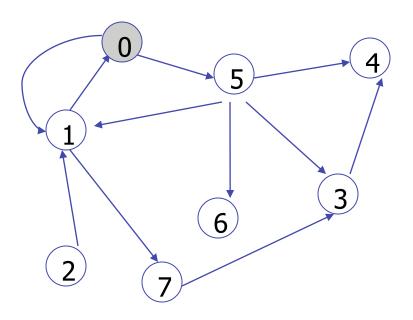
```
BFS (G, s)
```

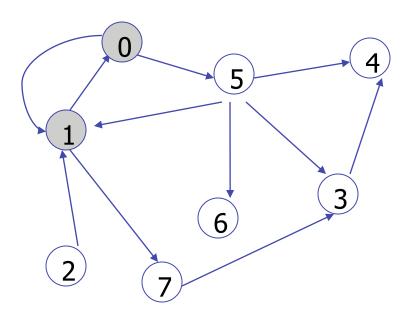
```
01 for each vertex u \in V[G] - \{s\} // anchor (s)
02
   colour[u] ← white // u colour
   d[u] \leftarrow \infty // s distance
0.3
04 \pi[u] \leftarrow NIL // u predecessor
05 \text{ colour[s]} \leftarrow \text{grey}
06 \, d[s] \leftarrow 0
07 \pi [s] \leftarrow NIL
08 <u>enqueue</u>(Q,s)
09 while Q \neq \emptyset do
10
   u ← <u>dequeue</u>(Q)
11
    for each v \in Adj[u] do
12
           If colour[v] = white then
13
               colour[v] ← grev
14
               d[v] \leftarrow d[u] + 1
15
              \pi[v] \leftarrow u
16
              enqueue (Q, v)
       colour[u] ← blue
```

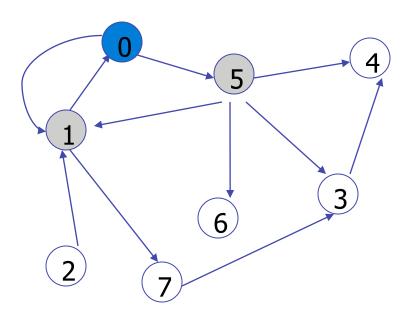
Initialization of graph nodes

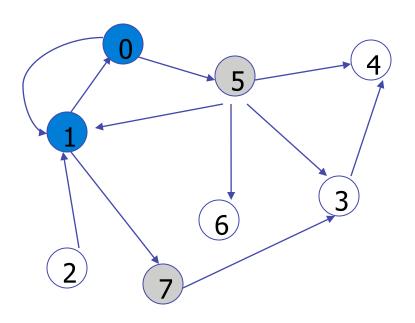
Initializes the anchor node (s)

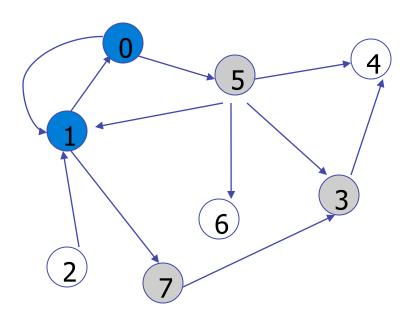
Visit each adjacent node of *u*

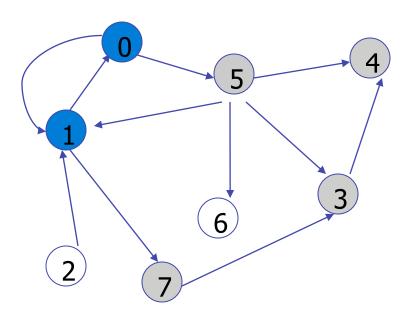


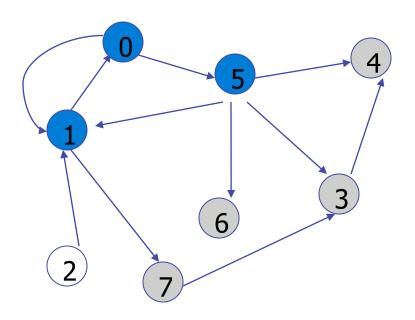


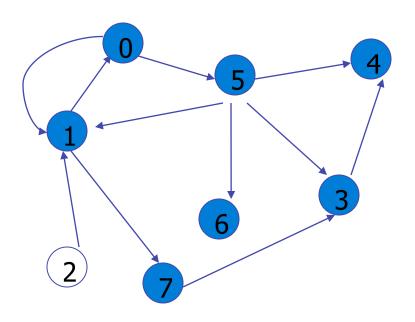




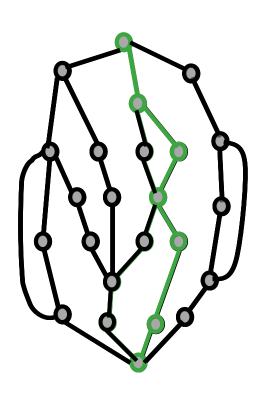








Symbolic Execution

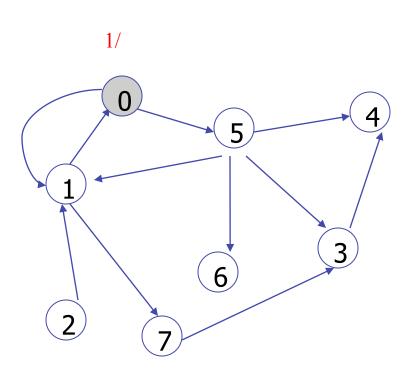


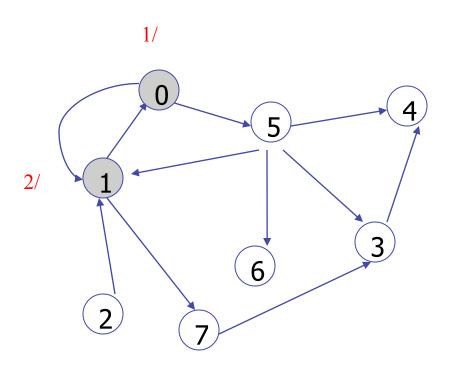
 Symbolic execution explores all paths individually

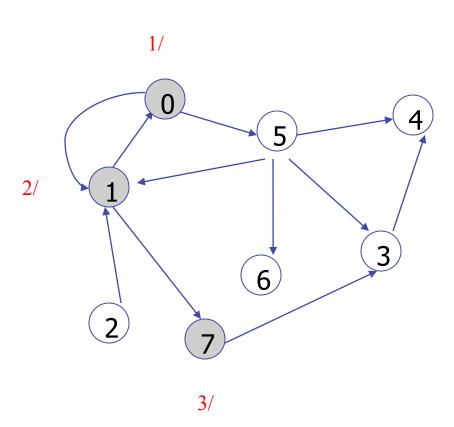
 Can only prove correctness if all paths are explored

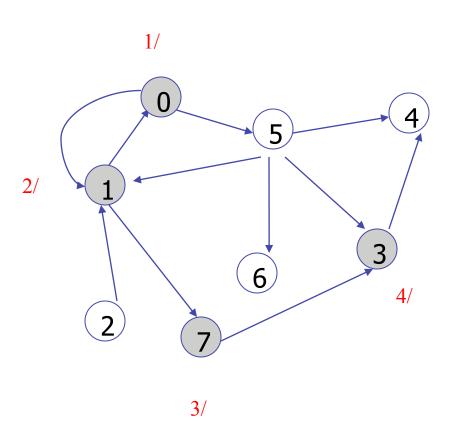
Depth-first search (DFS)

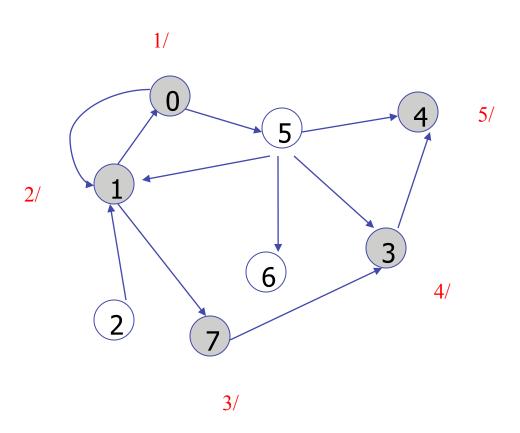
```
DFS(G)
                                                Paint all vertices white and
    for each vertex u \in V[G]
         do color[u] \leftarrow WHITE
                                                initialize the fields \pi with NIL
            \pi[u] \leftarrow \text{NIL}
                                                where \pi [u] represents the
4 time \leftarrow 0
5 for each vertex u \in V[G]
                                                predecessor of u
         do if color[u] = WHITE
6
               then DFS-VISIT(u)
DFS-VISIT(u)
   color[u] \leftarrow GRAY \triangleright White vertex u has just been discovered.
2 time \leftarrow time + 1
3 \quad d[u] \leftarrow time
    for each v \in Adj[u] \triangleright Explore edge (u, v).
5
         do if color[v] = WHITE
6
               then \pi[v] \leftarrow u
                     DFS-VISIT(v)
   color[u] \leftarrow BLACK  \triangleright Blacken u; it is finished.
   f[u] \leftarrow time \leftarrow time + 1
```

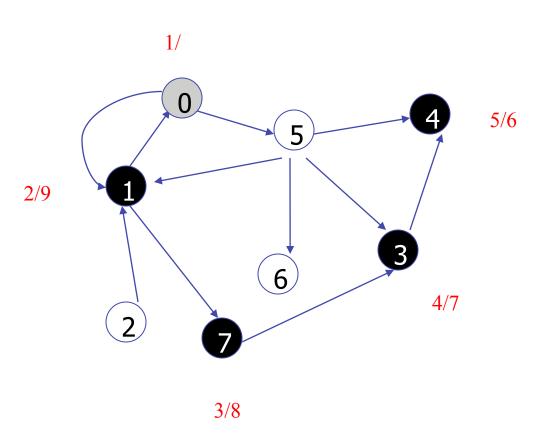


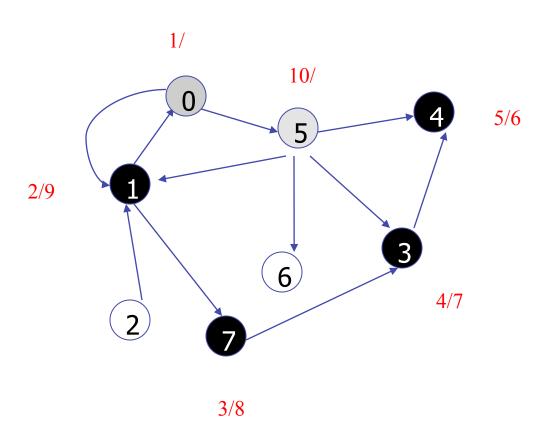


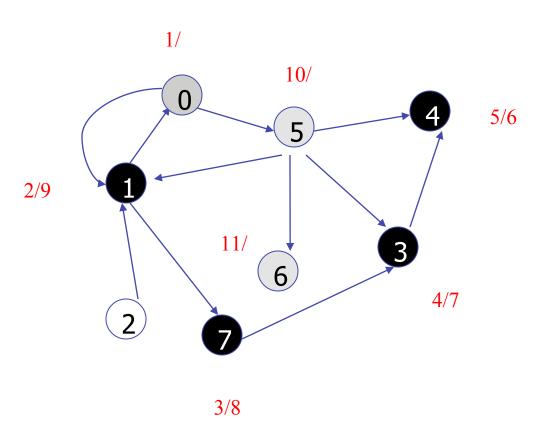


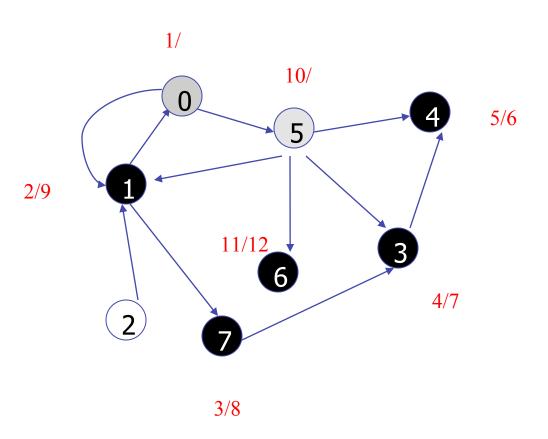


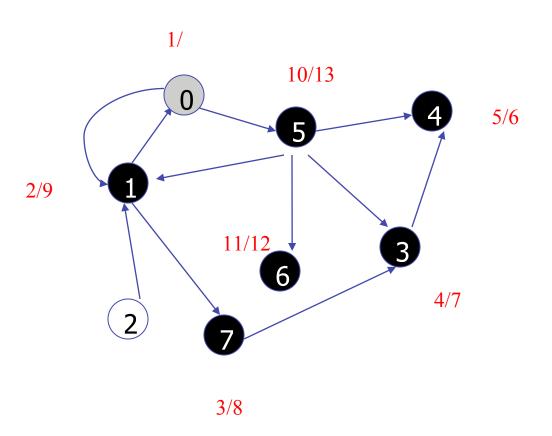


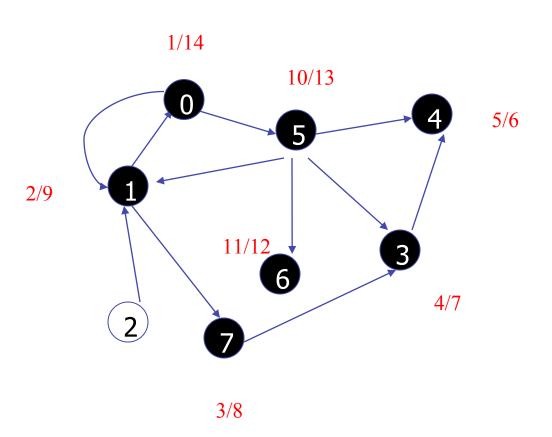


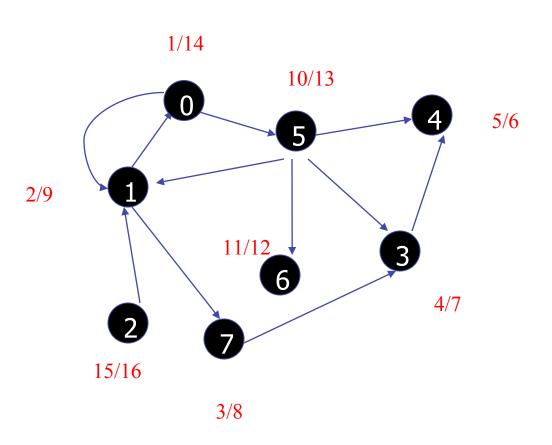












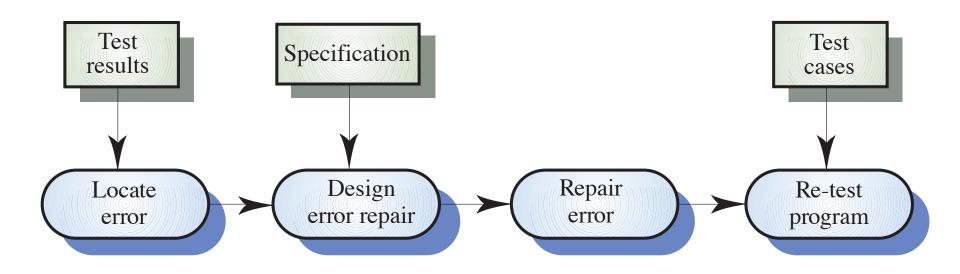
V & V and debugging are distinct processes

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 - Locating and
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- V & V is concerned with establishing the absence or existence of defects in a program, resp.
- Debugging is concerned with two main tasks
 - Locating and
 - Repairing these errors
- Debugging involves
 - Formulating a hypothesis about program behaviour
 - Test these hypotheses to find the system error

The debugging process



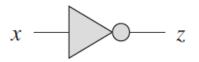
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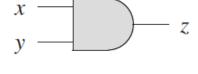
Intended learning outcomes

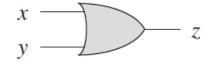
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 - Connectives: ∧ (AND), ∨ (OR), and ¬ (NOT)





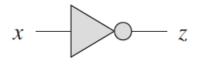


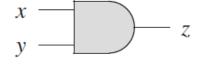
$\boldsymbol{\mathcal{X}}$	$\neg x$
0	1
1	0

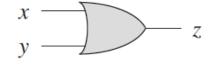
$\boldsymbol{\mathcal{X}}$	y	$x \wedge y$
0	0	0
0	1	0
1	0	0
1	1	1
		ı

$\boldsymbol{\mathcal{X}}$	y	$x \vee y$
0	0	0
0	1	1
1	0	1
1	1	1

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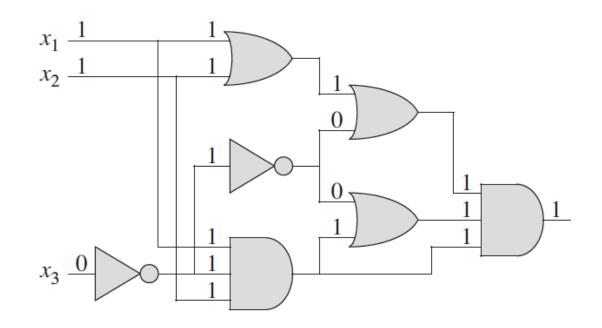
$\boldsymbol{\mathcal{X}}$	$\neg x$
0	1
1	0

х	y	$x \wedge y$
0	0	0
0	1	0
1	0	0
1	1	1

$$\begin{array}{c|cccc}
x & y & x \lor y \\
\hline
0 & 0 & 0 \\
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 1
\end{array}$$

 A Boolean formula is SAT if there exists some assignment to its variables that evaluates it to 1

 A Boolean combinational circuit consists of one or more Boolean combinational elements interconnected by wires



SAT:
$$\langle x_1 = 1, x_2 = 1, x_3 = 0 \rangle$$

Circuit-Satisfiability Problem

 Given a Boolean combinational circuit of AND, OR, and NOT gates, is it satisfiable?

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- Size: number of Boolean combinational elements plus the number of wires
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- When the size of C is polynomial in k, checking each one takes $\Omega(2^k)$
 - o Super-polynomial in the size of *k*

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Example:

$$o \Phi = ((x_1 \rightarrow x_2) \vee \neg ((\neg x_1 \leftrightarrow x_3) \vee x_4)) \wedge \neg x_2$$

o Assignment: $\langle x_1 = 0, x_2 = 0, x_3 = 1, x_4 = 1 \rangle$

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o
$$\Phi = ((0 \rightarrow 0) \lor \neg ((\neg 0 \leftrightarrow 1) \lor 1)) \land \neg 0$$

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$$\Phi = ((0 \rightarrow 0) \lor \neg ((\neg 0 \leftrightarrow 1) \lor 1)) \land \neg 0$$

$$0 \Phi = (1 \vee \neg (1 \vee 1)) \wedge 1$$

$$ο Φ = (1 \lor 0) \land 1$$

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$$\circ \Phi = ((\mathsf{x}_1 \to \mathsf{x}_2) \vee \neg ((\neg \mathsf{x}_1 \leftrightarrow \mathsf{x}_3) \vee \mathsf{x}_4)) \wedge \neg \mathsf{x}_2$$

o Assignment:
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$$0 \Phi = ((0 \rightarrow 0) \vee \neg ((\neg 0 \leftrightarrow 1) \vee 1)) \wedge \neg 0$$

$$0 \Phi = (1 \vee \neg (1 \vee 1)) \wedge 1$$

$$ο Φ = (1 \lor 0) \land 1$$

DPLL satisfiability solving

Given a Boolean formula φ in *clausal form* (an AND of ORs)

$$\{\{a, b\}, \{\neg a, b\}, \{a, \neg b\}, \{\neg a, \neg b\}\}$$

determine whether a *satisfying assignment* of variables to truth values exists.

DPLL satisfiability solving

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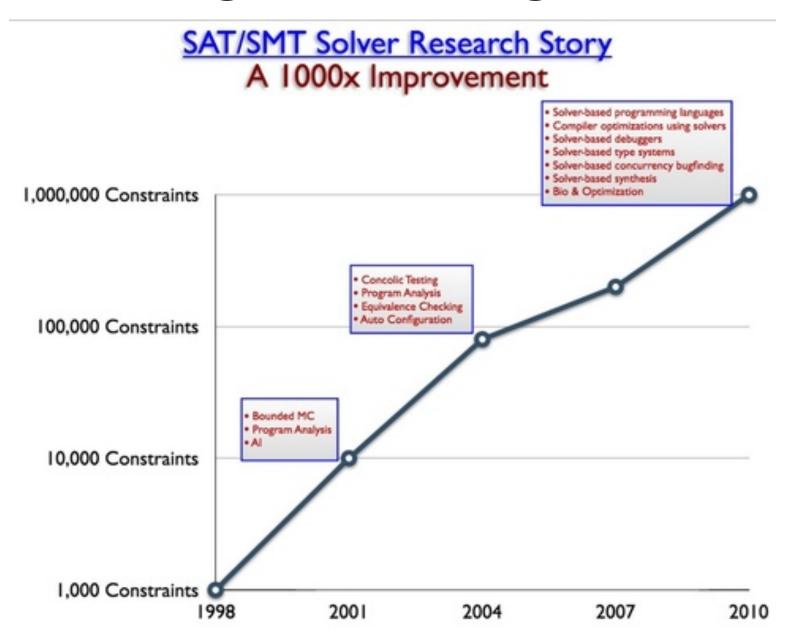
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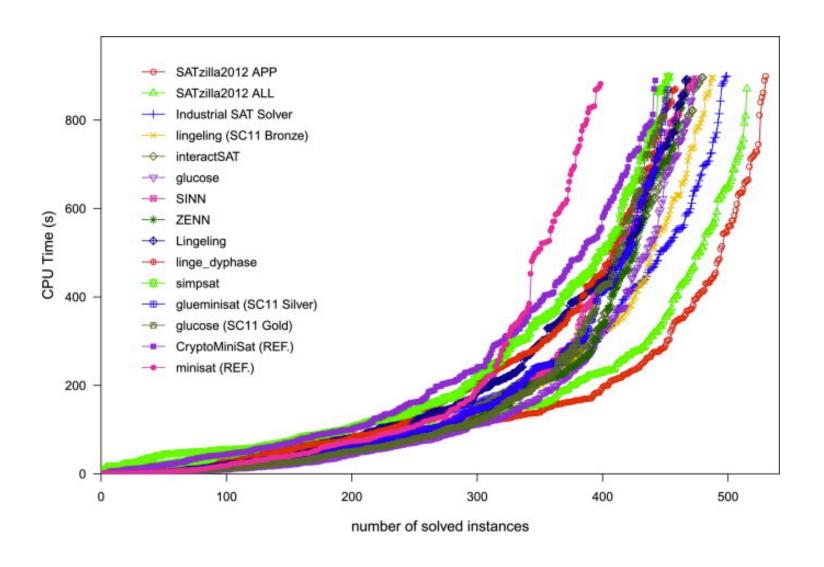
a → false

- 1. If $\varphi = \emptyset$ then SAT
- 2. If $\square \subseteq \varphi$ then UNSAT
- 3. If $\varphi = \varphi' \cup \{x\}$ then DPLL($\varphi'[x \mapsto true]$)
 If $\varphi = \varphi' \cup \{\neg x\}$ then DPLL($\varphi'[x \mapsto false]$)
- 4. Pick arbitrary x and return $b \mapsto false$ $b \mapsto true$ $b \mapsto true$ $b \mapsto true$ $b \mapsto true$ $b \mapsto true$
- + NP-complete but many heuristics and optimizations
- ⇒ can handle problems with 100,000's of variables

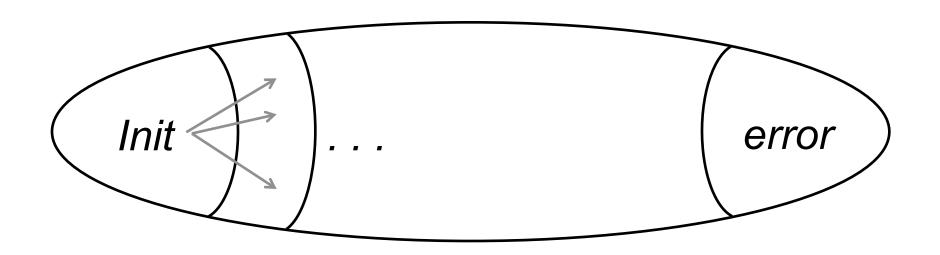
SAT solving as enabling technology



SAT Competition

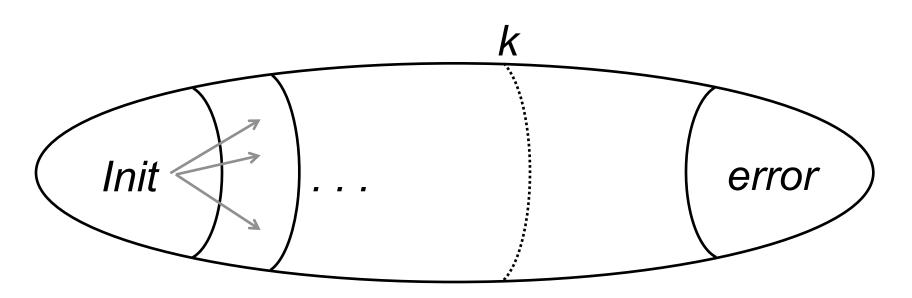


MC: check if a property holds for all states

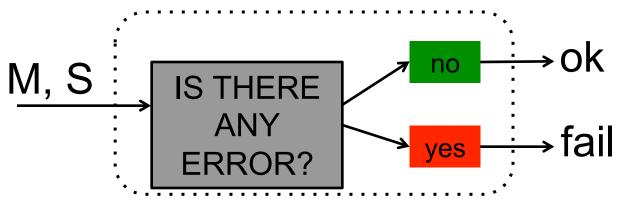


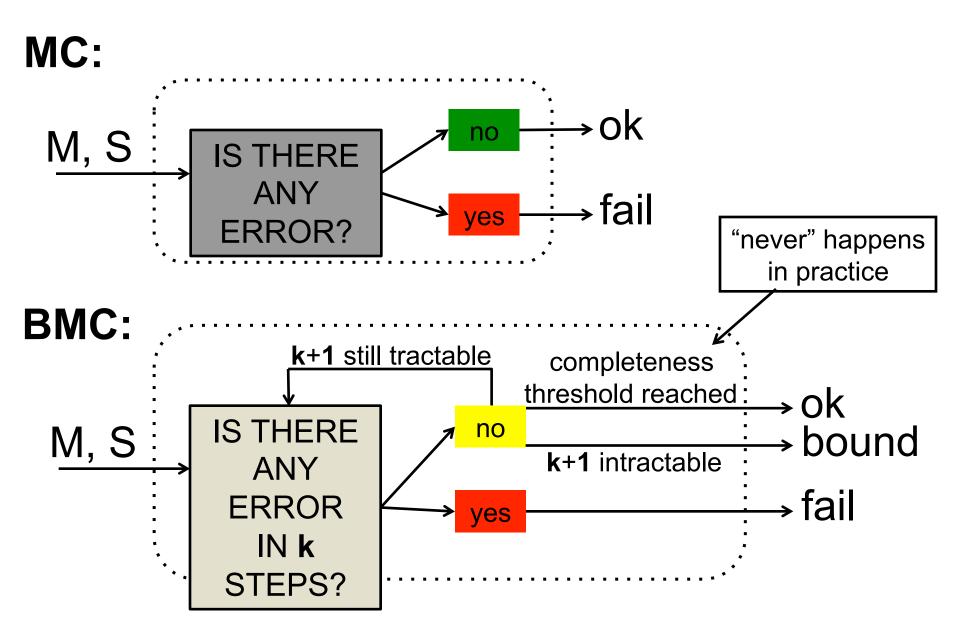
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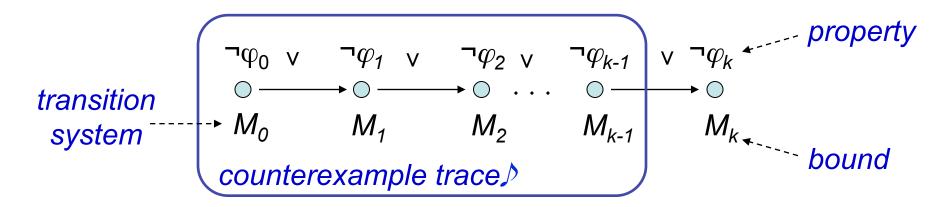
BMC: check if a property holds for a subset of states

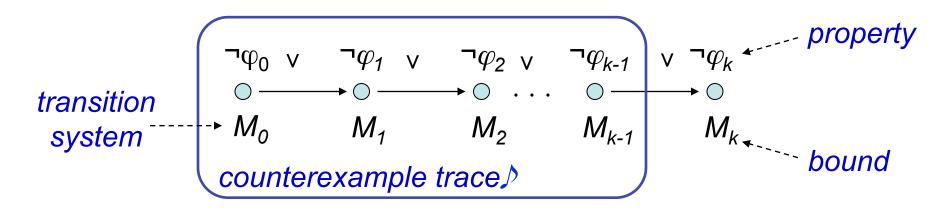


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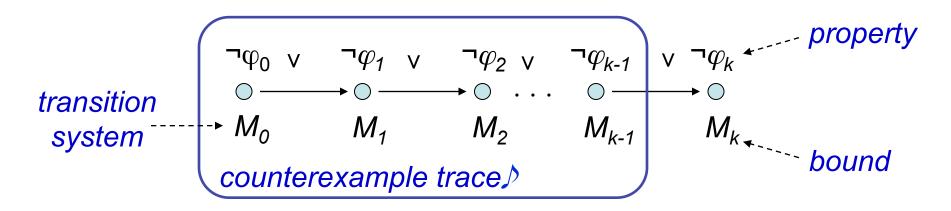




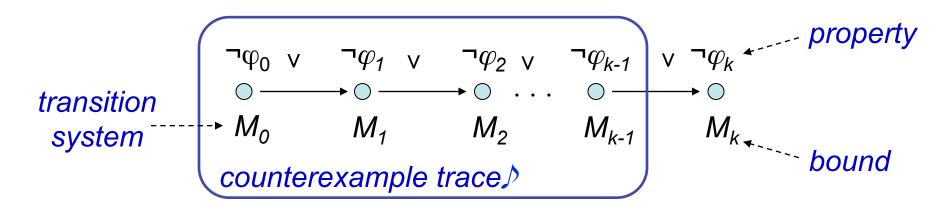




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- has been applied successfully to verify HW/SW systems

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using facts about bit-vector arithmetic

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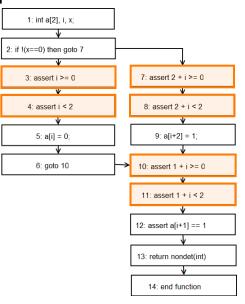
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- SMT solvers also apply:
 - standard algebraic reduction rules
 - contextual simplification

BMC of Software

- program modelled as state transition system
 - state: program counter and program variables
 - derived from control-flow graph
 - checked safety properties give extra nodes
- program unfolded up to given bounds
 - loop iterations
 - context switches
- unfolded program optimized to reduce blow-up
 - constant propagation crucial
 - forward substitutions

```
int main() {
  int a[2], i, x;
  if (x==0)
    a[i]=0;
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    a[i+2]=1;
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```
g_1 = x_1 == 0

a_1 = a_0 WITH [i_0:=0]

a_2 = a_0

a_3 = a_2 WITH [2+i_0:=1]

a_4 = g_1 ? a_1: a_3

t_1 = a_4[1+i_0] == 1
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- extraction of constraints C and properties P
 - specific to selected SMT solver, uses theories
- satisfiability check of C ∧ ¬P

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$$C := \begin{cases} g_1 := (x_1 = 0) \\ \land a_1 := store(a_0, i_0, 0) \\ \land a_2 := a_0 \\ \land a_3 := store(a_2, 2 + i_0, 1) \\ \land a_4 := ite(g_1, a_1, a_3) \end{cases}$$

$$P := \begin{bmatrix} i_0 \ge 0 \land i_0 < 2 \\ \land \ 2 + i_0 \ge 0 \land 2 + i_0 < 2 \\ \land \ 1 + i_0 \ge 0 \land 1 + i_0 < 2 \\ \land \ select(a_4, i_0 + 1) = 1 \end{bmatrix}$$

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- majority of VCs solved faster if numeric types are modelled by abstract domains but possible loss of precision
- ESBMC supports both types of encoding and also combines them to improve scalability and precision

Bitvector encodings need to handle

- type casts and implicit conversions
 - arithmetic conversions implemented using word-level functions (part of the bitvector theory: Extract, SignExt, ...)
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 - define error literals to detect over- / underflow for other types

```
res_op \Leftrightarrow \neg \text{ overflow}(x, y) \land \neg \text{ underflow}(x, y)
```

o similar to conversions

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Rational encoding: convert a to a rational number

$$a = \begin{cases} \left(i * p + \left(\frac{f * p}{2^n} + 1\right)\right) & \text{//} p = \text{number of decimal places} \\ p & \text{:} f \neq 0 \end{cases}$$

$$i : \text{otherwise}$$

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 - Five rounding modes: round nearest with ties choosing the even value, round nearest with ties choosing away from zero, round towards zero, round towards positive infinity and round towards negative infinity

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- Both solvers offer non-standard functions:
 - fp_as_ieeebv: converts floating-point to bitvectors
 - fp_from_ieeebv: converts bitvectors to floating-point

How to encode Floating-point programs?

- Most operations performed at program-level to encode
 FP numbers have a one-to-one conversion to SMT
- Special cases being casts to boolean types and the fp.eq operator
 - Usually, cast operations are encoded using extend/extract operation
 - Extending floating-point numbers is non-trivial because of the format

```
int main()
{
    _Bool c;

    double b = 0.0f;
    b = c;
    assert(b != 0.0f);

    c = b;
    assert(c != 0);
}
```

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Floating-point Encoding: Illustrative Example

```
int main()
  float x;
  float y = x;
  assert (x==y);
  return 0;
```

```
; declaration of x and y
(declare-fun |main::x| () (_ FloatingPoint 8 24))
(declare-fun |main::y| () (_ FloatingPoint 8 24))
; symbol created to represent a nondeteministic number
(declare-fun |nondet_symex::nondet0| () (_ FloatingPoint 8 24))
; Global guard, used for checking properties
(declare-fun |execution_statet::\\guard_exec| () Bool)
; assign the nondeterministic symbol to x
(assert (= |nondet_symex::nondet0| |main::x|))
; assign x to y
(assert (= |main::x| |main::y|))
; assert x == y
(assert (let ((a!1 (not (=> true
                    (=> |execution_statet::\\guard_exec|
                        (fp.eq |main::x| |main::y|)))))
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                                                -Assignment x to y
; assign x to y
(assert (= |main::x| |main::y|))
; assert x == y
(assert (let ((a!1 (not (=> true
                   (=> |execution_statet::\\guard_exec|
                       (fp.eq |main::x| |main::y|)))))
 (or a!1)))
```

```
; declaration of x and y
(declare-fun |main::x| () (_ FloatingPoint 8 24))
(declare-fun |main::y| () (_ FloatingPoint 8 24))
; symbol created to represent a nondeteministic number
(declare-fun |nondet_symex::nondet0| () (_ FloatingPoint 8 24))
; Global guard, used for checking properties
(declare-fun |execution_statet::\\guard_exec| () Bool)
; assign the nondeterministic symbol to x
                                             Check if the comparison
(assert (= |nondet_symex::nondet0| |main::x|))
                                                 satisfies the guard
; assign x to y
(assert (= |main::x| |main::y|))
; assert x == y
(assert (let ((a!1 (not (=> true
                   (=> |execution_statet::\\guard_exec|
```

(fp.eq |main::x| |main::y|)))))

(or a!1)))

Z3 produces:

```
sat
(model
  (define-fun |main::x| () (_ FloatingPoint 8 24)
        (_ NaN 8 24))
  (define-fun |main::y| () (_ FloatingPoint 8 24)
        (_ NaN 8 24))
  (define-fun |nondet_symex::nondet0| () (_ FloatingPoint 8 24)
        (_ NaN 8 24))
  (define-fun |execution_statet::\\\guard_exec| () Bool
        true)
)
```

MathSAT produces:

```
sat
( (|main::x| (_ NaN 8 24))
   (|main::y| (_ NaN 8 24))
   (|nondet_symex::nondet0| (_ NaN 8 24))
   (|execution_statet::\\guard_exec| true) )
```

```
Counterexample:
State 1 file main3.c line 3 function main thread 0
main
 State 2 file main3.c line 4 function main thread 0
main
 State 3 file main3.c line 5 function main thread 0
main
Violated property:
 file main3.c line 5 function main
 assertion
 (Bool)(x == y)
VERIFICATION FAILED
```

Intended learning outcomes

- Introduce software verification and validation
- Understand soundness and completeness concerning detection techniques
- Emphasize the difference among static analysis, testing / simulation, and debugging
- Explain bounded model checking of software
- Explain precise memory model for software verification

- arrays and records / tuples typically handled directly by SMT-solver
- pointers modelled as tuples

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```
int main() {
  int a[2], i, x, *p;
  p=a;
  if (x==0)
   a[i]=0;
  else
   a[i+1]=1;
  assert(*(p+2)==1);
}
```

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  int a[2], i, x, *p;
  p=a;
  if (x==0)
    a[i]=0;
  else
    a[i+1]=1;
  assert(*(p+2)==1);
}
(p_1 := store(p_0, 0, &a[0]) \\ \land p_2 := store(p_1, 1, 0) \\ \land g_2 := (x_2 == 0) \\ \land a_1 := store(a_0, i_0, 0) \\ \land a_2 := a_0 \\ \land a_3 := store(a_2, 1+ i_0, 1) \\ \land a_4 := ite(g_1, a_1, a_3) \\ \land p_3 := store(p_2, 1, select(p_2, 1)+2))
```

- arrays and records / tuples typically handled directly by SMT-solver
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Store object at position 0

```
p_1 := store(p_0, 0, &a[0])
int main() {
                                       \land p_2 := store(p_1, 1, 0)
 int a[2], i, x, *p;
                                       \land g_2 := (x_2 == 0)
 p=a;
                                      \land a_1 := store(a_0, i_0, 0)
 if (x==0)
                                      \wedge a_2 := a_0
   a[i]=0;
                                       \land a_3 := store(a_2, 1 + i_0, 1)
 else
                                       \land a_4 := ite(g_1, a_1, a_3)
   a[i+1]=1;
                                       \land p_3 := store(p_2, 1, select(p_2, 1)+2)
 assert(*(p+2)==1);
```

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Store object at position 0

```
p_1 := store(p_0, 0, &a[0])
int main() {
                                     \land p_2 := store(p_1, 1, 0)
 int a[2], i, x, *p;
                                     \wedge g_2 := (x_2 = 2)
                                     \land a_1 := store(a_0, i) Store index at
 p=a;
 if (x==0)
                                                             position 1
                                     \wedge a_2 := a_0
   a[i]=0;
                                      \land a_3 := store(a_2, 1 + i_0, 1)
 else
                                     \land a_4 := ite(g_1, a_1, a_3)
   a[i+1]=1;
                                      \land p_3 := store(p_2, 1, select(p_2, 1)+2)
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int main() {
                                          \land p_2 := store(p_1, 1, 0)
 int a[2], i, x, *p;
                                          \wedge g_2 := (x_2 = 2)
                                          \land a_1 := store(a_0, i) Store index at
 p=a;
 if (x==0)
                                                                        position 1
                                      Update index
   a[i]=0;
                                          rac{1}{\sqrt{a_3}} e(a_2, 1+ i_0, 1)
 else
                                         \land a_4 := \text{ite}(g_1, a_1, a_3)

\land p_3 := \text{store}(p_2, 1, \text{select}(p_2, 1) + 2)
   a[i+1]=1;
 assert(*(p+2)==1);
```

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- model memory just as an array of bytes (array theories)
 - read and write operations to the memory array on the logic level

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 - $-\upsilon$ \triangleq indicate whether the object is still alive

- model memory just as an array of bytes (array theories)
 - read and write operations to the memory array on the logic level
- each dynamic object d_o consists of

 - $-\rho \triangleq unique identifier$
 - $-\upsilon$ \triangleq indicate whether the object is still alive
- to detect invalid reads/writes, we check whether
 - d_o is a dynamic object
 - i is within the bounds of the memory array

$$l_{is_dynamic_object} \Leftrightarrow \left(\bigvee_{j=1}^{k} d_o.\rho = j\right) \land \left(0 \le i < n\right)$$

- to check for invalid objects, we
 - set υ to true if the function malloc can allocate memory (d_o is alive)
 - set υ to *false* if the function free is called (d_o is not longer alive)

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$$I_{valid_object} \Leftrightarrow (I_{is_dynamic_object} \Rightarrow d_o.v)$$

- to detect forgotten memory, at the end of the (unrolled) program we check
 - whether the d_o has been deallocated by the function free

$$I_{\text{deallocated object}} \Leftrightarrow (I_{\text{is dynamic object}} \Rightarrow \neg d_o.v)$$

```
#include <stdlib.h>
void main() {
    char *p = malloc(5);  // ρ = 1
    char *q = malloc(5);  // ρ = 2
    p = q;
    free(p)
    p = malloc(5);  // ρ = 3
    free(p)
}
```

Assume that the malloc call succeeds

```
#include <stdlib.h>
void main() {
  char *p = malloc(5); // \rho = 1
  char *q = malloc(5); // \rho = 2
P:= (\neg d_{o1}.v \land \neg d_{o2}.v \neg d_{o3}.v)
  p=q;
  free(p)
  p = malloc(5); 	 // \rho = 3
  free(p)
         \begin{pmatrix} d_{o1}.\rho=1 \ \land \ d_{o1}.s=5 \ \land \ d_{o1}.\upsilon=true \ \land \ p=d_{o1} \\ \land \ d_{o2}.\rho=2 \ \land \ d_{o2}.s=5 \ \land \ d_{o2}.\upsilon=true \ \land \ q=d_{o2} \end{pmatrix} 
C:= \bigwedge p=d_{o2} \land d_{o2}.v=false

\bigwedge d_{o3}.\rho=3 \land d_{o3}.s=5 \land d_{o3}.v=true \land p=d_{o3}

\bigwedge d_{o3}.v=false
```

```
#include <stdlib.h>
void main() {
  char *p = malloc(5); // \rho = 1
  char *q = malloc(5); // \rho = 2
P:= (\neg d_{o1} \cdot v \land \neg d_{o2} \cdot v \neg d_{o3} \cdot v)
  p=q;
  free(p)
  p = malloc(5); 	 // \rho = 3
  free(p)
        \begin{pmatrix} d_{o1}.\rho=1 \ \land \ d_{o1}.s=5 \ \land \ \mathbf{d_{o1}.}v=\mathbf{true} \ \land \ p=d_{o1} \\ \land \ d_{o2}.\rho=2 \ \land \ d_{o2}.s=5 \ \land \ d_{o2}.v=\mathbf{true} \ \land \ q=d_{o2} \end{pmatrix} 
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\bigwedge d_{o3}.v=false
```

Align-guaranteed memory mode

- Alignment rules require that any pointer variable must be aligned to at least the alignment of the pointer type
 - E.g., an integer pointer's value must be aligned to at least
 4 bytes, for 32-bit integers

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 - E.g., an integer pointer's value must be aligned to at least
 4 bytes, for 32-bit integers
- Encode property assertions when dereferences occur during symbolic execution
 - To guard against executions where an unaligned pointer is dereferenced
 - This is not as strong as the C standard requirement, that a pointer variable may never hold an unaligned value
 - o But it provides a guarantee that any pointer dereference will either be correctly aligned or result in a verification failure

- statically tracks possible pointer variable targets (objects)
 - dereferencing a pointer leads to the construction of guarded references to each potential target

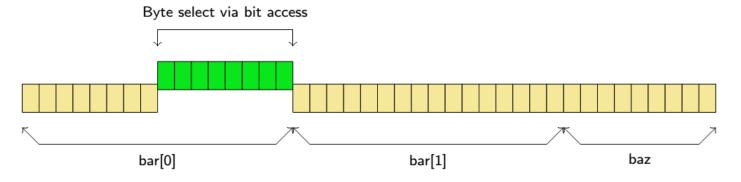
- statically tracks possible pointer variable targets (objects)
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- C is very liberal about permitted dereferences

```
struct foo {
    uint16_t bar[2];
    uint8_t baz;
};
struct foo qux;
char *quux = &qux;
quux++;
    pointer and object types
do not match
```

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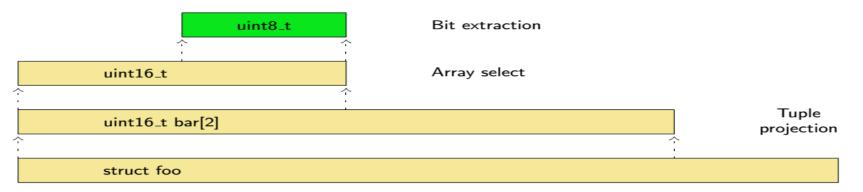
SAT: immediate access to bit-level representation



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SMT: sorts must be repeatedly unwrapped



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 - requires manipulation of arrays / tuples
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- supporting all legal behaviors at SMT layer difficult
 - extract (unaligned) 16bit integer from *fuzz
- experiments showed significantly increased memory consumption

 framework cannot easily be changed to SMT-level byte representation (a la LLBMC)

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- push unwrapping of SMT data structures to dereference
- enforce C alignment rules
 - static analysis of pointer alignment eliminates need to encode unaligned data accesses
 - → reduces number of behaviors that must be modeled
 - add alignment assertions (if static analysis not conclusive)
 - extracting 16-bit integer from *fuzz if guard is true:
 - offset = 0: project bar[0] out of foo
 - offset = 1: "unaligned memory access" failure
 - offset = 2: project bar[1] out of foo
 - offset = 3: "unaligned memory access" failure
 - offset = 4: "access to object out of bounds" failure

Summary

- Described the difference between soundness and completeness concerning detection techniques
 - False positive and false negative
- Pointed out the difference between static analysis and testing / simulation
 - hybrid combination of static and dynamic analysis techniques to achieve a good trade-off between soundness and completeness
- Explained bounded model checking of software
 - they have been applied successfully to verify singlethreaded software using a precise memory model